ABSTRACT

Numerical Optimization of the Filter Function used in the Convolution Filtered Back Projection Algorithm for Computed Tomography

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The work described here addresses the problem of choosing the optimum filter function for use in convolution filtered back projection. The choice of the filter function has a major effect on the amount of photon noise in reconstructed images, on the amount and type of artefact present, and on the resolution. Since many of the factors affecting artefact and resolution in practical reconstructions are difficult to take into account analytically, and/or depend on difficult to measure properties of specific machines, there is need for an automatic numerical procedure for choosing the filter function.

A numerical algorithm capable of calculating the filter function that produces the best picture as judged by a wide variety of criteria has been developed. A simple measure of image quality which it can optimize is the sum of the squares of the differences between the reconstructed pixels and their "ideal" values. This sum can be minimized in the presence of noise for any reconstructed

object, or its average for any collection of objects can be minimized. More generally, any second degree function of the reconstructed pixels, including cross terms, can be minimized.

The key to the algorithm is a new analysis of the effect of interpolation during back projection on the reconstructed image. This interpolation, which seems unavoidable in a computationally practical reconstruction algorithm, causes the reconstructed image to be a linear combination of a very finite set of "standard" reconstructions, where the coefficients in the linear combination are determined by and determine the filter function. Any quadratic function of the image is therefore also a quadratic function of these coefficients, and so the values of the coefficients which minimize it can easily be found. The coefficients then determine the filter function.

The optimization algorithm has been used to calculate a filter function which decreases artefact in a computer simulated reconstruction by a factor of more than three, reduces photon noise power by a factor of about two, and causes no apparent decrease in effective resolution; all relative to Shepp and Logan's filter function. These improvements apparently result from taking proper account of aliasing artefact.

It is anticipated that a major application of the algorithm will be to the reduction of artefacts specific to particular tomographic scanners. These artefacts can result from collimation difficulties, use of fan beams, non.linearities, and other problems.