Adaptive Spectral Strain Estimators for Elastography

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In conventional elastography, internal tissue deformations, induced by external compression applied to the tissue surface, are estimated by cross-correlation analysis of echo signals obtained before and after compression. Conventionally, strains are estimated by computing the gradient of estimated displacement. However, gradient-based algorithms are highly susceptible to noise and decorrelation, which could limit their utility. We previously developed strain estimators based on a frequency-domain (spectral) formulation that were shown to be more robust but less precise compared to conventional strain estimators. In this paper, we introduce a novel spectral strain estimator that estimates local strain by maximizing the correlation between the spectra of pre- and postcompression echo signals using iterative frequency-scaling of the latter; we also discuss a variation of this algorithm that may be computationally more efficient but less precise. The adaptive spectral strain estimator combines the advantages of time- and frequency-domain methods and has outperformed conventional estimators in experiments and 2-D finite-element simulations.

KEY WORDS: Correlation; elastogram; elastography; imaging; spectral estimation; spectral shift; strain; stress; ultrasound; ultrasonic imaging.

I. INTRODUCTION

This paper describes a novel, spectral, strain-estimation approach. Tissue elasticity can be altered by pathological tissue changes and many cancers are typically stiffer than the surrounding tissue. Physicians routinely use palpation to detect tissue abnormalities such as cancer in the breast, prostate, and other organs. However, use of palpation is limited to shallow lesions and/or lesions that are significantly stiffer than their surroundings. Palpation also cannot provide quantitative information and its efficacy depends on physician skill and experience. Ultrasound-based elasticity imaging methods have been developed to overcome these limitations and they provide a palpation-like ability to evaluate internal structures. While conventional B-mode sonograms convey information regarding acoustic-scattering properties, elasticity images convey information regarding tissue elasticity, primarily Young’s modulus. Elasticity and sonographic parameters usually are unrelated so that elastograms contain information unavailable in sonograms. Elasticity imaging methods enjoy the advantage of the inherently large dynamic range in tissue elasticity; differences in tis-
sue elasticity can be a few orders of magnitude in the presence of abnormalities,\textsuperscript{1,16} as opposed to differences of only a few percent typical in other imaging modalities. This sensitivity to tissue type has motivated widespread investigations of elasticity imaging.

Reviews of ultrasound-based elasticity imaging can be found in references 13-15. These methods comprise two broad categories: (1) imaging of tissue response to external or internal quasi-static loading or compression\textsuperscript{9,10} and (2) imaging of tissue response to external dynamic loading or vibration,\textsuperscript{3,8} which typically use Doppler-type methods to sense internal motion. Compression-based methods or elastography, the method addressed in this article, produce high-resolution elastograms (elastographic images) that quantitatively depict local tissue deformation. Initial clinical elastographic studies for breast-cancer detection are very encouraging.\textsuperscript{11}

Strain is usually estimated by computing the local gradient of displacement resulting from quasi-static external compression. Local tissue displacements are estimated using cross-correlation analysis between the pre- and postcompression echo segments. An elastogram is formed by estimating strain throughout a tissue cross-section by segmenting echoes along each scan line into overlapping temporal windows.

In the presence of large, irregular and non-axial tissue motions, major echo-signal decorrelations occur and cause conventional strain estimators to fail. Various strain-estimation strategies have been developed to facilitate strain estimation in harsh signal environments. The least-squares strain estimator proposed by Kallel and Ophir\textsuperscript{17} uses a multi-point linear regression rather than a two-point gradient operation that tends to amplify the noise in displacement estimates. Applied compression itself causes decorrelation, which global stretching\textsuperscript{18,19} was found to reduce. An adaptive stretching algorithm\textsuperscript{20} estimates strain using an iterative approach that varies the local stretch factor to maximize the correlation between pre- and postcompression echo signal segments. Local strain was observed to cause a scaling of rf echo spectra that is proportional to strain, which can be measured via the spectral centroid shift\textsuperscript{21} or spectral cross-correlation.\textsuperscript{22} These noncoherent estimators estimate strain directly (i.e., without a gradient operation) within a single estimation window (as in adaptive stretching\textsuperscript{20}) and are less precise but more robust than commonly-used coherent (cross-correlation) methods.

In this paper, we describe a new spectral adaptive scaling method where the principle of adaptive stretching has been applied in the spectral domain combining their advantages. This method is based on the echo spectrum; the postcompression spectrum is frequency-scaled until its similarity with the precompression spectrum is maximized. Two procedures are described for maximizing this similarity. We first illustrate the methods using 1-D simulations. Then, we evaluate the spectral strain estimators using 2-D finite element simulations for a wide range of applied strains. Finally, we have employed experimental data from an elasticity phantom subjected to an external compression. In our study, spectral methods have outperformed the conventional strain estimators and provide us with alternate, superior strain estimators.

II. METHODS

A. Spectral strain estimation

For our analysis, we use a simplified one-dimensional (1-D) model. In this case, the pre- and postcompression echo signals can be expressed as:

\begin{equation}
    r_i(t) = b(t) * h(t) + n_i(t)
\end{equation}
where \( b(t) \) is the effective backscatter distribution in 1-D (this is a spatial function, converted into a function of time using \( t = 2x/c \), where \( x = \) distance from transducer and \( c = \) propagation speed), \( h(t) \) is the impulse response of the ultrasonic system, \( n_1(t) \) and \( n_2(t) \) are uncorrelated random noise and * denotes convolution. The parameter \( a = 1 - \varepsilon \), where \( \varepsilon \) is strain (assumed constant). The compression of \( b(t) \) appears as a depth-dependent displacement when observed under a small data window at small strains (\( \varepsilon < 1 \)). This makes it possible to estimate strain by computing the gradient of displacement estimates. We have ignored additive noise rest of the way for simplified analysis. Using the convolution and time-scale properties of the Fourier Transform,

\[
R_r(f) = B(f)H(f)
\]

(2a)

\[
R_r(f) = |a|B(af)H(f)
\]

(2b)

where uppercase symbols are used to denote transformed functions.

By time-scaling (or temporally stretching) the postcompression signal by a factor \( \alpha_r \) we obtain

\[
r_r(t) = r_r(\alpha_r,t) = b(\alpha_r t / a) \ast h(\alpha_r t)
\]

(3a)

In a similar manner, frequency-scaling (compression) the postcompression complex spectrum by a factor \( \alpha_r \) yields

\[
R_r(f) = R_r(f / \alpha_r) = |a|B(af / \alpha_r)H(f / \alpha_r)
\]

(3b)

Appropriate time-scaling (i.e., \( a_1 = a \)) improves correlation between pre- and postcompression echo signals.\(^{17-19}\) Furthermore, correlation between \( r_1(t) \) and \( r_3(t) \) is maximum when \( a_3 = a \).

This fact motivated the development of an adaptive time-domain method that estimates local strain by computing the time-scale factor that maximizes correlation between \( r_1(t) \) and \( r_3(t) \).\(^{20}\) Similarly, Eq. (3b) implies that the correlation between \( R_1(f) \) and \( R_3(f) \) is maximum when \( a_3 = a \). This suggests the utility of a frequency-domain strain estimator, which iteratively scales the postcompression echo spectrum \( R_r(f) \) by a varying factor \( \alpha_r \). The scale factor that maximizes correlation between the power spectra \( |R_1(f)|^2 \) and \( |R_3(f)|^2 \) characterizes the corresponding strain itself. Note that time or frequency scaling is not exact because it also erroneously scales the system response, which is unaffected by compression. At lower strains, this is generally not an issue because \( B(f) \) varies rapidly compared to \( H(f) \); at higher strains, however, scaling in time or frequency can encounter problems.

We illustrate the above concepts using a 1-D simulation. The round-trip impulse response \( h(t) \) has a center frequency of 5 MHz and a 60% bandwidth. Figure 1a shows 1.5 mm seg-

\(^{(1)}\)In conventional elastography, applied strain has typically been limited to 1%. At higher strains, decorrelation between \( r_1(t) \) and \( r_3(t) \) becomes substantial (due to nonaxial motion, interference, etc., which are unavoidable in practice), introducing significant error in the strain estimation. In ideal 1-D simulations, only 1-D effects are present. Thus, considerably lower decorrelations occur in 1-D simulations than in practice. 1-D simulations were thus used only for illustration (Figs. 1–3, 5–7) in this paper. Our results show that despite the simplified model, the method works in practical situations.
ments of pre- and postcompression radiofrequency (rf) echo signals for a uniform strain of 2% \( (e = 0.02) \). The postcompression signal \( r_2(t) \) closely resembles the precompression signal \( r_1(t) \) compressed 2%. \( r_1(t) \) is then time-scaled ("temporal stretching") by the same factor scatterers were compressed, producing \( r_3(t) \). \( r_1(t) \) and \( r_3(t) \) are very similar but not identical because the embedded system transfer function \( h(t) \) is also time-scaled (Eq. (3a)).

Figure 1b shows the associated pre- and postcompression echo power spectra, \( |R_1(f)|^2 \) and \( |R_2(f)|^2 \), of 3 mm segments. As expected from Eq. (2b), we observe an upward frequency shift of 2.04% \( [1/(1 - 0.02) - 1] \) in the postcompression spectrum over the signal bandwidth. As per Eq. (3b), we frequency-scaled the postcompression power spectrum \( |R_2(f)|^2 \) to compensate for the effect of strain. \( |R_1(f)|^2 \) and \( |R_3(f)|^2 \) are nearly identical at lower frequencies. However, \( |R_3(f)|^2 \) tails off faster at higher frequencies; compression of the postcompression spectrum also compresses the transfer function \( H(f) \), causing \( |R_3(f)|^2 \) to decrease more rapidly at high frequencies (Eq. (3b)).

Figures 2 and 3 show the simulated echo signals and their power spectra at uniform strains of 1% and 4%. At low strains, postcompression signals resemble the precompression signals compressed by a factor equaling the applied strain. When strain increases, the similarity of postcompression signals and precompression signals is reduced and time-scaling cannot fully restore the signal shape. In the frequency domain, strain induces an upward frequency shift in postcompression spectra, which increases with increasing strain. Spectral shapes of \( R_1(f) \) and \( R_3(f) \) almost completely agree at low frequencies if \( \alpha_1 = \alpha \); \( |R_3(f)|^2 \) tails off more quickly at higher frequencies due to scaling of \( H(f) \). This effect is more pronounced as strains become higher due to the increased scaling of \( H(f) \).

To illustrate the proposed spectral estimator, we first computed the power spectra of 3 mm segments of simulated (1-D) pre- and postcompression echo signals at 2% uniform strain. The postcompression echo power spectra \( |R_2(f)|^2 \) were then frequency-scaled using variable factors corresponding to strain values from 0 to 4%. Finally, the correlation (normalized) between \( |R_1(f)|^2 \) and \( |R_3(f)|^2 \) were computed and the results were plotted in figure 4a. As expected, the correlation is maximum when the frequency-scale factor matches true strain of 2%; thus, the frequency-scale factor that maximizes the correlation between the pre- and postcompression spectra appears to be feasible for estimating strain. We have varied the applied strain, repeated the procedure and plotted the estimated strains and the corresponding maximum correlation values vs. true strains in figure 4b. In these examples, this method estimates strain accurately at all strains, where conventional estimators are known to fail at higher strains. The attainable correlation decreases with increasing strain. These cases are
FIG. 2 1-D examples of time-scaling (top) and frequency-scaling (bottom) for 1% applied strain.

FIG. 3 1-D examples of time-scaling (top) and frequency-scaling (bottom) for 4% applied strain.
only illustrative examples; as noted in footnote (1), we expect the estimators to perform worse in practical applications, which includes effects not included in 1-D simulations.

In the time-domain, nonzero time-delay between pre- and postcompression signal segments usually is present because of tissue motion. Thus, to find the stretch factor that maximizes correlation between pre- and postcompression signal segments, a correlation search needs to be performed at each stretch factor. However, in the spectral domain, such delays are generally not encountered because a shift in time results in a linear phase shift vs. frequency. Thus, a correlation search is generally not necessary to find the correlation maximum in the frequency-domain, which is a key difference between this method and adaptive stretching. As shown in figures 1–3, the echo power spectra fall within the same frequency band and we need to compute the correlation between pre- and postcompression spectra at zero lag only.

The flow chart in figure 5a illustrates the estimation of strain for the spectral algorithm at one data window location. To compute a full elastogram, this process is repeated at all windows. The shaded box (choosing the next value of $\alpha$) is a critical component in this algorithm. For this purpose, we typically use a ‘binary search’ method, illustrated using a flow-chart in figure 5b. At any time, two stretch factors are under consideration. Of two current stretch factors, the one producing the larger correlation is retained and the search interval is halved, until a stopping criterion is satisfied. To reduce the possibility of locking onto a false maximum, an initial coarse search using a few equally-spaced scale factors is generally performed; a binary search is then invoked. Other search algorithms such as hierarchical search may also be used.

We considered an alternate and potentially more efficient method, which computes strain by minimizing the variance of the ratio

$$\frac{|R_s(f)|^2}{|R_s(f)|^2} = \left[\frac{\alpha / a}{\alpha / a} B(a f / \alpha / a) H(f / \alpha / a)\right]^2$$

(4)

When $\alpha = a$,\n
$$\frac{|R_s(f)|^2}{|R_s(f)|^2} = \left[\frac{H(f / a)}{H(f)}\right]^2$$

(5)
The ratio in Eq. (5) contains contributions only from the smoothly-varying transfer function. When $\alpha \neq a$, the ratio in Eq. (4) will exhibit rapid variations vs. $f$ due to contributions from rapidly varying $B(f)$. Thus, local strain may be estimated by minimizing the deviation in this ratio from smooth behavior over the signal bandwidth. We have plotted this ratio for no scaling ($\alpha = 1$) and appropriate scaling ($\alpha = a$) vs. frequency for 2% strain (Fig. 6a). As expected, when $\alpha = a$, the spectral ratio varies smoothly with frequency. We then have frequency-scaled the postcompression echo spectra using varying factors corresponding to 0 to 4% strains and have computed the variance of ratio between $R_i(f)$ and $R_j(f)$ (approximate frequency band used: 4–6 MHz) and have plotted the results in figure 6b. The variance is minimum when the frequency-scale factor matches the true strain (2%); thus, the frequency-scale factor that minimizes the variance of the spectral ratio in Eq. (4) can be used for estimating strain. Figure 7 illustrates the performance of this approach for varying strains and shows the estimated strains and the corresponding minimum variances vs. true strains. This estima-

FIG. 5 (a) Flow-chart representation of adaptive spectral scaling. (b) Binary search method. Initial values of $\alpha_1$ and $\alpha_2$ can be obtained using a coarse search.

FIG. 6 (a) Spectral ratio in Eq. (4) for no scaling (solid, —) and proper scaling (dashed, ---). At proper scale factor, it varies smoothly with frequency. Applied strain is 2%. (b) Variance of spectral ratio vs. scale-factor (or assumed strain). Variance is minimum when the assumed strain matches true strain.

![Graphs and figures illustrating the adaptive spectral strain estimation](image-url)
tor works reasonably well at all strains, including higher strains where conventional methods could fail. The results also show that the minimum variance increases with increasing strain. This approach includes a division; thus, care must be taken to assure that the denominator does not contain zeroes or very small values. This condition can usually be satisfied by evaluating the ratio Eq. (4) only within the signal bandwidth. (A 15 dB bandwidth has proven useful.) In addition, $|R_1(f)|^2$ can be inspected for occasional very small values, which can then be discarded. The deviation in this ratio from smooth behavior can be estimated approximately from its variance. However, even a smooth variation will cause the variance of this ratio to be nonzero. A preferred approach is to first smooth the ratio, subtract the smoothed function from the original ratio and then compute the variance of the result.

This algorithm is nearly identical to the one shown in figure 5a, the only difference being that a variance of a ratio is minimized rather than a correlation being maximized. Furthermore, the notch near the minimum ratio is significantly sharper than the peak of the correlation-based approach, which could make this estimator more precise. However, a sharp extremum is also easier to miss.

Adaptive spectral scaling combines the advantage of locally-varying scaling with that of frequency-domain formulation. The frequency-domain formulation has several advantages over its time-domain counterpart. It involves a product of functions as opposed to a convolution and rf echo signals are quickly varying functions of time, whereas their spectra are slowly varying functions of frequency. Thus, we expect adaptive spectral scaling to be more robust but slightly less precise.

### B. 2-D finite element simulation and phantom experiments

We have tested the spectral estimators with data generated using 2-D finite element analysis and phantom experiments. The simulations treated and employed a rectangular phantom was simulated using the finite element (FEM) analysis software ALGOR™ (Algor, Inc., Pittsburgh, PA). This phantom had dimensions of 40 mm × 40 mm and contained four circular inclusions each of 7.5 mm diameter (Fig. 8a), with a homogeneous background stiffness

![FIG. 7 Estimated strain (top) and corresponding variance (bottom) vs. true strain for spectral ratio method.](image-url)
of 60 kPa (close to the average stiffness of normal glandular tissue in the breast). An incompressible material was simulated (Poisson’s ratio, \(\nu = 0.495\)). The top left, bottom, top right and middle inclusions were 3.16, 10, 31.62, and 100 times (10, 20, 30, and 40 dB) stiffer than the background, respectively. The bottom of the phantom was constrained in the vertical direction and compressed from the top by a compressor that was wider than the phantom. The phantom was scanned with an ultrasonic transducer from the top (center frequency = 5 MHz, fractional bandwidth = 60%) and was allowed to freely expand both at the top and bottom. A nondiffracting Gaussian beam\(^{(2)}\) with 1.5 mm diameter was simulated and the total number of A-lines was 60. Random white noise was added to simulate a sonographic SNR of 40 dB; in many applications including the breast and prostate, an SNR of 30-40 dB is typical. Figure 8b shows an ideal elastogram (simulation how tissue will deform under uniform compression) for a 2% applied strain. Although the background has uniform stiffness, strain variations occur in the background due to interaction between the stiffer lesions.

We digitally acquired rf echo-signal data for elasticity phantom experiments using an ATL (Bothell, WA) Ultramark 9 scanner and L10-5 (7.5 MHz) linear array transducer; a Spectrasonics Inc. (Wayne, PA) acquisition module was interfaced with the scanner. Data were sampled at 20 MHz at an effective dynamic range of 14 bits. The data were interpolated to achieve an effective sampling rate of 50 MHz prior to processing. Time-gain-control (TGC) data were acquired for every scan and the rf data were corrected for TGC before processing. A single transmit focal zone and dynamic receive focusing was employed. The custom-made phantom, supplied by CIRS, Inc. (Norfolk, VA) had dimensions of 90 mm × 90 mm × 120 mm and included a stiff (3 times stiffer) cylindrical inclusion (of 2 cm diameter) in an otherwise homogeneous surrounding. The scan plane was perpendicular to the inclusion axis so the inclusion would appear circular in B-mode images and elastogram. The phantom’s acoustic properties are representative of soft tissue, with a speed of sound in the 1540 m/s range. Rf echo signals were acquired before and after compression of the phantom.

\(^{(2)}\)It can be contemplated/hypothesized as a beam with very large depth-of-focus; thus, decorrelation does not vary with depth. Although it is not entirely practical, close approximation is possible at least for a finite depth.\(^{24, 25}\)
with a custom-made bench-top computer-controlled compression fixture, which holds the phantom between two horizontal plates; the top plate includes a hole for placing the transducer. A compressor large enough to cover the entire phantom was used to produce relatively uniform stress conditions in the phantom. All processing software was written in MATLAB™ (The Mathworks, Inc., Natick, MA).

III. RESULTS

Figure 9 compares the performance of spectral estimators with that of conventional estimators and adaptive stretching at applied strains of 1–8% for the FEM simulation described in the previous section. For processing, we used a data window size of 3 mm and an inter-window shift of 0.5 mm. To reduce any ‘salt and pepper’ noise, a $3 \times 3$ median filter was used on computed elastograms. Ideal elastograms at applied strains of 1%, 2%, 4% and 8% are shown in figure 9a. Figure 9b shows the corresponding elastograms for global uniform stretching (strain computed using least squares strain estimator). These elastograms appear satisfactory at low applied strains. However, some very noticeable artifacts (unnatural strain-variation inside the lesions) are visible; these artifacts increase with increasing applied strain and are particularly apparent in the top and the bottom left lesions. The elastograms have significant noise in the areas around the lesions at higher applied strains. Global stretching itself introduces decorrelations in the low strain regions by overstretching the post compression signal; this effect worsens as the contrast between the lesion and the background increases, especially at higher applied strains. Figure 9c shows elastograms processed using adaptive stretching, which are significantly less noisy. All the lesions are clearly visible and the difference between four lesions is obvious. At higher applied strains, noise levels in the elastograms increase but are much lower than those in Figure 9b. Even at 8% applied strain, the elastogram shows all salient phantom features, despite significant noise around the lesions. Figure 9d shows elastograms for adaptive spectral scaling. The results appear very similar to that in figure 9c; all lesions are clearly visible. However, the elastograms are marginally noisier overall compared to figure 9c. Figure 9e shows elastograms for spectral ratio estimator. These elastograms are much noisier than the elastograms in figure 9c. However, the image noise was considerably less dependent on applied strain. The area around the lesions appears to the cleanest of all methods at higher strains.

We quantitatively assess and compare the performance of different methods by evaluating the similarity of computed elastograms to the ideal elastogram (Fig. 10). We have computed the rms error between the ideal and the estimated elastograms after image registration. (A higher rms error would signify worse performance.) For all the methods, performance degrades at higher strain, as expected. However, performance of the conventional method degrades faster than other methods with increasing applied strain. A daptive stretching and adaptive spectral scaling perform significantly better at all applied compressions followed by the spectral ratio method. (Adaptive spectral scaling marginally outperforms adaptive stretching.) We note that the rms error might underestimate image degradation because it is evaluated for the entire elastogram while errors may preferentially occur at specific locations.

To test the limits of spectral estimators in following rapidly varying strains, we simulated strain profiles resembling fm chirps. We simulated 25 A-lines, each with identical cyclic strain profile of more than 25 cycles. (Maximum strain = 4%, minimum strain = 0% and mean strain = 2%). The rate of change of strain increases linearly with depth. Farthest from the transducer, the spatial period of the strain sinusoid is less than 1 mm. The psf center fre-
frequency was 5 MHz and the bandwidth was 60%. To adequately sample the strain variation at all depth, we used a 1 mm window with 88% window overlap. Figure 11 shows the performance of adaptive stretching compared to that of the proposed spectral methods; the adaptive stretching method follows the true strain profile quite well only in the first half cycle. As the rate of change of strain increases, both techniques begin to show increased difficulty in following the strain profile, especially near the maxima and the minima. We have previ-

**FIG. 9** Performance of various strain estimators for a phantom simulated with 2-D finite element analysis. (a) Ideal elastogram, (b) Global uniform stretching (with least squares), (c) adaptive stretching, (d) adaptive spectral scaling, and (e) spectral ratio.
ously shown that adaptive stretching significantly outperforms conventional methods in following rapidly-varying strain.

Experimental phantom data were also used to evaluate performance of spectral estimators. We computed elastograms for experimental data from an inhomogeneous phantom described in the previous section. A 3 mm correlation window with 75% window overlap was used. We used a 3 × 3 median filter on the elastograms to reduce ‘salt and pepper’ noise. The elastograms are shown in figure 12 for applied strain of 4%. Figure 12a shows the elastogram for conventional estimator. There is significant noise at both corners at the top. Inside the lesion is also noisy. Figure 12b shows the elastogram for adaptive stretching. There is much less noise at top corners and the lesion is much more clearly visible. Figure

FIG. 10 FEM simulation: inhomogeneous phantom. Comparison of performance of different methods by evaluating the rms errors between the computed and ideal elastograms.

FIG. 11 Ability of the estimator to follow rapidly varying strain using 1-D simulation for chirp strain profile. The solid line (—) shows the simulated strain and the dashed line (---) shows estimated strain. (a) Adaptive stretching. (b) Adaptive spectral scaling. (c) Spectral ratio.
IV. DISCUSSION

In this paper, we describe novel elastographic strain estimators that employ spectral processing in order to overcome limitations of conventional elasticity imaging methods. The current practice of conventional elastography generally requires a relatively large computer-controlled tissue-compression fixture to avoid large and irregular tissue motions that can cause major echo-signal decorrelations. Conventional strain estimators fail in the presence of major decorrelations. Even when compression fixtures are used, echo-signal decorrelations occur from nonaxial tissue motion and from tissue compression itself that causes signal shape to change. Various approaches, developed to overcome these problems, include global uniform stretching\textsuperscript{18,19} to reduce decorrelation from axial tissue compression and a least-squares approach\textsuperscript{17} to reduce noise associated with gradient operations. Global stretching only compensates for tissue compression on an average, whereas the proper stretch factor is dependent on locally-varying strain. Based on this observation, we previously developed an adaptive stretching strain estimator\textsuperscript{20} that estimates the proper stretch factor (thereby strain itself) locally by iteratively maximizing the correlation between pre- and postcompression echo signal segments. Later, we developed spectral strain estimators\textsuperscript{21,22} and found them to be less precise but more robust than conventional strain estimators; these estimators were based on the shift of power spectra as a result of strain. We wanted to combine the advantages of adaptive stretching and the early spectral estimators by developing an adaptive frequency-scaling algorithm. We anticipated these estimators would be quite precise while retaining robustness in the presence of large strains and undesirable motions. The spectral estimators proposed in this paper performed well in the FEM simulations and phantom experiments at large applied strains. Interestingly, previously developed, robust, adaptive stretching (time-domain) performed similarly at large strains.

This strain estimator, like any 1-D estimator, makes the following assumptions that are not completely realistic — plane wave insonation, only-axial motion, noise-free echo data, constant strain over the window and a psf that is invariant with depth (linear shift invariant imaging systems). However, despite ignoring these issues, the estimator is shown to perform well at low-to-moderate applied strains for FEM simulated as well as experimental data. For larger strains, nonaxial motion will be considerable. Spectral strain estimators, in their pres-
ent implementation, cannot reduce the decorrelation from nonaxial scatterer movements. However, there is no fundamental reason why a search in the lateral direction cannot also be incorporated in the algorithm; this will, of course, increase computational complexity. To estimate out-of-plane motion for this and other estimators, true 2D arrays are necessary.

Unlike experimental data, the 2-D FEM simulations do not include scatterer motion in the elevational direction. However, the ultrasonic beam in 2-D array transducers is significantly wider in elevational direction than lateral (azimuthal) direction. Thus, scatterer movement in the elevational direction is only a secondary source of decorrelation and its absence does not make the simulation unrealistic.

Estimating the correlation maximum is time-consuming. We can use a SAD (sub-absolute-difference)\(^6\) or SSD (sum-squared-difference) and minimize it instead of maximizing correlation. SAD is defined as

\[
\varepsilon_{xy}[i] = \sum_{k=0}^{N-1} |x[k] - y[k+i]|
\]  

(6)

SSD is defined as

\[
\varepsilon_{xy}[i] = \sum_{k=0}^{N-1} (x[k] - y[k+i])^2
\]  

(7)

SAD and SSD searches are much more time-efficient at some reduction of performance. Note that SAD and SSD searches do not handle changes in signal amplitudes very well.

Choosing a proper data window size is important because elastographic resolution\(^{27,28}\) is related to it. (Smaller windows typically yield better resolution.) Our earlier experience showed that for high bandwidth transducers with 5–7.5 MHz center frequencies, window sizes significantly less than 2 mm might produce noisy images for time-domain strain estimators.\(^3\) (We used 3 mm window sizes in this paper.) In contrast, significant strain variations may occur within the data window at large window sizes, which may introduce additional signal decorrelation; all strain estimators implicitly assume constant strain within the window. Obviously, an optimum exists between the two extremes. The optimal window size, of course, also depends on various system and processing parameters, e.g., local strain, center frequency, bandwidth, SNR, etc.

Estimators based on time-domain correlations can accommodate the ubiquitous delay between pre- and postcompression signal segments. In contrast, these delays result in linearly varying phase terms in the frequency domain as scatterers move in and out of data windows (pre- and postcompression data segments may not contain the same scatterers); this effect can change the power spectra in a complicated and sometimes unpredictable fashion that causes problems for spectral strain estimators. We have aligned the data window of postcompression echo signals with that of precompression echo signals to partially overcome this limitation. For a uniform strain \(\varepsilon\), the precompression signal at a depth \(d\) is aligned with the postcompression signal at a depth \(d(1 - \varepsilon)\). In this implementation, the postcompression window starts at \(w_t(1 - \varepsilon)\) if the precompression window starts at \(w_t\) for an applied strain of \(\varepsilon\). However, local strains generally are spatially variable; thus, pre- and postcompression signal segments will not be fully aligned. An additional iteration that applies small variation in the postcompression window location around \(w_t(1 - \varepsilon)\) to maximize the correlation between pre- and postcompression spectra has been employed.

\(^{3}\) There is a trade-off between noise and resolution. Larger windows improves SNR; smaller windows improves resolution.
Although frequency-scaling methods apply to tissue spectra, they also scale the system transfer function $H(f)$, which is unaffected by strain. This artifactually changes the relative amplitudes of spectral peaks, particularly at large strain values (e.g., as shown in figures 1–3). If the distortion of $H(f)$ causes significant errors in strain estimation, an estimate of $H(f)$ can be included in the estimation algorithm. $H(f)$ can also be estimated from local power spectra estimates over small regions-of-interest. Alternatively, because precompression spectrum and frequency-scaled postcompression spectrum are nearly identical at lower frequencies (the difference increases with increasing frequency), only the lower frequency range can be used for the estimation, especially in the presence of a large strain. If time-domain methods are to be used low-pass filtering of the rf data in the presence of moderate irregular or large motion may be improve performance.

To gain insight whether the performance of adaptive spectral scaling is due to phase information discarded in the power spectra, we investigated whether discarding phase in the time domain results in similar performance. Figure 13 shows elastograms computed using the envelope of rf echo signals, rather than the rf echo signals for the FEM simulated data used to generate figure 9. (Figure 9a shows the ideal elastograms at applied strains of 1%, 2%, 4% and 8%.) Figure 13a repeats the elastograms shown in figure 9d for adaptive spectral scaling. Figure 13b shows the corresponding elastograms for global uniform stretching (envelope). It performs better than their rf counterpart but worse than adaptive spectral scaling. Figure 13c shows the elastograms computed using time-domain adaptive stretching of envelope signals; surprisingly, these elastograms perform significantly worse than their rf coun-
ter parts. Clearly, adaptive spectral scaling outperforms the two time domain methods where phase was discarded and its superior performance is not simply due to phase being discarded.

In conclusion, spectral domain adaptive scaling provides superior performance compared to conventional time-domain methods. However, time-domain adaptive stretching also provides similar performance and thus, because of the large computation time, frequency-scaling is indicated only when adequate computation power is available. The spectral methods described in this article have a moderate-to-high computational complexity; computational time is at least one order of magnitude greater than that of conventional methods.

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REFERENCES


**APPENDIX**

Normalized correlation function between the power spectra of precompression and frequency-scaled postcompression echo signals can be expressed as

\[ \rho_{11}(\phi) = \frac{\int_{-\infty}^{\infty} P_1(f)P_3(f + \phi)df}{\sqrt{\int_{-\infty}^{\infty} P_1^2(f)df \int_{-\infty}^{\infty} P_3^2(f + \phi)df}} \quad (A1) \]

Here \( P \) denotes power spectra, so that \( P_3(f) = |R_3(f)|^2 \).

Using Eqs. (A1), (2a) and (3b) and ignoring noise terms,

\[ \rho_{11}(\phi) = \frac{\int_{-\infty}^{\infty} |B(f)H(f)|^2 \left\{ \frac{a}{\alpha_f} B \left( \frac{a}{\alpha_f} (f + \phi) \right) H \left( \frac{1}{\alpha_f} (f + \phi) \right) \right\}^2 df}{\sqrt{\int_{-\infty}^{\infty} |B(f)H(f)|^4 df \int_{-\infty}^{\infty} \left\{ \frac{a}{\alpha_f} B \left( \frac{a}{\alpha_f} (f + \phi) \right) H \left( \frac{1}{\alpha_f} (f + \phi) \right) \right\}^4 df}} \]

or,

\[ \rho_{11}(\phi) = \frac{\int_{-\infty}^{\infty} |B(f)H(f)|^2 \left\{ B \left( \frac{a}{\alpha_f} (f + \phi) \right) H \left( \frac{1}{\alpha_f} (f + \phi) \right) \right\}^2 df}{\sqrt{\int_{-\infty}^{\infty} |B(f)H(f)|^4 df \int_{-\infty}^{\infty} \left\{ B \left( \frac{a}{\alpha_f} (f + \phi) \right) H \left( \frac{1}{\alpha_f} (f + \phi) \right) \right\}^4 df}} \quad (A2) \]
The well-known Schwartz’s inequality for continuous frequency-domain real functions $G_1(f)$ and $G_2(f)$ is expressed as

If $\int_{-\infty}^{\infty}|G_1(f)|^2 \, df < \infty$ and $\int_{-\infty}^{\infty}|G_2(f)|^2 \, df < \infty$,

then

$$\left| \int G_1(f)G_2^*(f) \, df \right|^2 \leq \int |G_1(f)|^2 \, df \int |G_2(f)|^2 \, df.$$ \hfill (4)

Thus,

$$\left| \int G_1(f)G_2^*(f) \, df \right|^2 \leq 1$$

$$\frac{\int |G_1(f)|^2 \, df \int |G_2(f)|^2 \, df}{\int |G_1(f)|^2 \, df \int |G_2(f)|^2 \, df}$$

or,

$$\frac{\int G_1(f)G_2^*(f) \, df}{\sqrt{\int |G_1(f)|^2 \, df \int |G_2(f)|^2 \, df}} \leq 1$$ \hfill (A3)

If we choose

$$G_1(f) = |B(f)H(f)|^2$$

and

$$G_2(f) = \left| B\left(\frac{a}{\alpha_f}\right)H\left(\frac{1}{\alpha_f}\right) \right|^2$$

then

$$\rho_{\text{rms}} = \frac{\int |B(f)H(f)|^2 \left| B\left(\frac{a}{\alpha_f}\right)H\left(\frac{1}{\alpha_f}\right) \right|^2 \, df}{\sqrt{\int |B(f)H(f)|^4 \, df \int \left| B\left(\frac{a}{\alpha_f}\right)H\left(\frac{1}{\alpha_f}\right) \right|^4 \, df}} \leq 1 \hfill (A4)$$

(4) For real signals, the equality is satisfied when $G_1(f) = kG_2(f)$, $k$ being a constant.
In earlier work, we have shown that time-scaling of the postcompression signal by the appropriate factor maximizes the correlation between the pre- and postcompression echo signals. Now, we postulate that proper frequency-scaling of the postcompression echo spectrum will maximize its correlation with the precompression spectrum. The improvement in correlation that results from frequency-scaling of the postcompression echo spectrum occurs due to the effective realignment of the scatterers; \( B(f) \) is a quickly-varying function of frequency whereas \( H(f) \) is a slowly-varying function of frequency. Thus, frequency-scaling of the system transfer function has little effect at low strain in Eq. (A4). Accordingly, we ignore frequency-scaling of the system transfer function and write \( H(f/a_f) \approx H(f) \) for \( a_f \approx 1 \).

Now we can write Eq. (A4) as

\[
\rho_{13_{\text{max}}} = \frac{\int |B(f)H(f)|^2 \left| B\left(\frac{a_f}{f}\right)H(f) \right|^2 df}{\left(\int |B(f)H(f)|^4 df\right)^{1/2}} \leq 1
\]

The maximum value of \( \rho_{13_{\text{max}}} \) to unity, when

\[
|B(f)H(f)|^2 = k|B\left(\frac{a_f}{a_f}f\right)H(f)|^2
\]

which clearly happens only if \( a_f = a \). (In fact, the maximum value is dependent on applied strain, \( \varepsilon \), which decreases with increasing strain from unity that occurs at \( \varepsilon = 0 \).) Thus, we can estimate local strain by iteratively frequency-scaling the postcompression echo spectra and maximizing the correlation between the precompression and postompression echo spectra.