Ultrasound Attenuation Imaging Using Compound Acquisition and Processing

HAIFENG TU, TOMY VARGHESE, ERNEST L. MADSEN, QUAN CHEN, AND JAMES A. ZAGZEBSKI

1Department of Medical Physics
2Department of Biomedical Engineering
The University of Wisconsin-Madison
Madison, WI 53706
htu@wisc.edu

A method that combines both spatial and frequency compounding is described for measuring attenuation in tissue. The technique applies a reference phantom to account for imaging system dependencies of echo signals. Emphasis is given to local attenuation estimates, to reduce the variance of the attenuation measurements over small regions of interest (ROI) and to enable coarse attenuation imaging. Experiments using a uniform phantom show that the standard deviation of local attenuation estimates within a ROI drops when greater degrees of compounding are applied. Attenuation images of a specially designed phantom containing inclusions with attenuation contrast illustrate the accuracy and precision of the technique.

KEY WORDS: Attenuation; compounding; frequency compounding; imaging; spatial compounding; ultrasound.

INTRODUCTION

Quantitative methods that enhance diagnostic capabilities of medical ultrasound instruments have received significant attention over the past three decades. Researchers have proposed approaches to measure first and second order statistical properties of echo signals, the scatter number density, the mean scatter spacing, the autocorrelation function and the scatter size. In recent years, ultrasound based elasticity imaging has also been investigated as a method that expands the role of this modality.

This paper describes a method for measurement and display of acoustic attenuation from pulse-echo data. Objectively-determined attenuation values could provide valuable information in assessing various forms of diffuse and focal liver disease, distinguishing ischemic from normal zones of the myocardium, differentiating between uterine fibroids and adenomyosis, and diagnosing breast masses. ‘Shadow’ signs on B-mode images are already used extensively to assess the likelihood of a malignant breast tumor. However, estimates of the actual ultrasound attenuation within such masses and correlation of results with pathology has not been done. Our motivation is to provide a means of measuring attenuation within small, localized regions viewed on ultrasound B-mode images and to produce attenuation images.

Many methods based on pulse echo techniques have been proposed for estimating the ultrasound attenuation coefficient. In general, they can be classified as either time domain or frequency domain methods. In the frequency domain, Kuc and Schwartz estimated attenuation using the slope of the difference between the logarithms of the echo signal power spectra from two different tissue depths. They developed a maximum likelihood estimator based on this method and measured attenuation of liver in vitro. Cloostermans et al proposed a
multi-narrowband method in which the slope of attenuation vs. frequency was derived through changes of the echo signal power spectra vs. depth, using short time Fourier analysis. Yao et al.\textsuperscript{17} further developed the idea by proposing a reference phantom method to account for machine and transducer beam dependencies.

Fink et al.\textsuperscript{18} estimated frequency-dependent attenuation using down-shifts of the centroid of the echo signal power spectrum acquired from increased depths. Kasai et al.\textsuperscript{19} proposed a centroid estimator using the first autocorrelation lag of the phase of the complex echo signal. Another centroid estimator was proposed by Kuc and Li\textsuperscript{20} using a second-order autoregressive model, and this technique was further developed by Baldeweck et al.\textsuperscript{21} Yet another frequency domain method is the matched filter, pulse compression approach developed by Meyer,\textsuperscript{22} said to be capable of providing results that are independent of overlapping echo wavetrains from adjacent tissue regions separated by $2/\Delta f$, where $\Delta f$ is the system bandwidth.

In the time domain, Flax et al.\textsuperscript{23} proposed a zero crossing method, which is a counterpart of the centroid shift method. Later, an ‘envelope peak’ method was proposed by He and Greenleaf,\textsuperscript{24} which uses the local maximum of the echo signal envelope in the attenuation estimation. Jang et al.\textsuperscript{25} estimated attenuation by measuring the entropy difference between two adjacent segments of envelope-detected, narrowband echo signals. Walach et al.\textsuperscript{26} produced one of the first ultrasound attenuation images, yielding a resolution of about 1.6 cm, utilizing the extended prony method. Knipp et al.\textsuperscript{27} developed the video signal analysis method to estimate the attenuation directly from B-mode images.

Except for work by Walach et al.\textsuperscript{26}, ‘local attenuation estimates’, that is, attenuation within small, selected regions of the scanned field, have received little attention by researchers, partially because of the requirement for large data segments to achieve statistically-accurate results. Here we present a method that decreases errors of attenuation estimations made over small volumes of tissue. The method uses spatial and frequency compounding in the data acquisition and analysis, estimating attenuation for the same volume but from statistically-independent signal samples. Experiments using a uniform phantom show that the standard deviation of local attenuation estimates drops when greater degrees of compounding are applied. Coarse attenuation images of a specially-designed attenuation phantom illustrate accuracy and precision of the technique.

**METHODS**

**A. Reference phantom method (RPM)**

It is natural to consider examining the loss of echo signal intensity or amplitude with depth to calculate the attenuation. However, beam diffraction and machine-dependent factors, such as transmit focus, gain, etc., need to be accounted for to make accurate and comparable attenuation estimations. The reference phantom method developed by Yao et al.\textsuperscript{17} is effective for accurate attenuation and backscatter estimations. In this method, the echo signal intensity from a sample is compared to the signal intensity at the same depth from a reference phantom, whose attenuation and backscatter properties are known. The reference phantom data are acquired using the same transducer and system settings used for acquiring echo data from the sample. After bandpass filtering, $r_i(f, z)$, the ratio of the echo signal intensity from the sample to that from the reference at frequency $f$ and depth $z$, can be expressed as \textsuperscript{17}

$$
ri(f, z) = \frac{i_s(f, z)}{i_r(f, z)} = \frac{\eta_s(f)}{\eta_r(f)} e^{-\alpha_s(f) - \alpha_r(f)z}
$$

(1)
where $i$ is the echo signal intensity; subscripts $s$ and $r$ refer to the sample and reference, respectively; the $\eta$’s are backscatter coefficients; and $\alpha$’s are sample and reference phantom attenuation coefficients at $f$.

After a least squares line-fit of the curve $\ln(r_i(f,z))$ vs. depth, $z$, the slope is proportional to the difference between the sample and reference attenuation coefficients. Since the latter is known, this provides the attenuation coefficient of the sample. For a uniform sample, the zero depth intercept yields the ratio of the sample and reference backscatter coefficients.

**B. Spatial and frequency compounding**

There are two types of uncertainty associated with these attenuation estimations. One is methodological, contributed by, for example, instrumental inaccuracy, nonuniformities in the medium and electronic noise. Another is due to the presence of statistical fluctuations in the backscattered signals from tissue. It is this latter statistical uncertainty that we address in this paper.

An analysis of the statistical uncertainty of attenuation values derived using the reference phantom method has been presented by Yao et al.\textsuperscript{28} Results show that $\sigma_a$, the standard deviation of the sample attenuation coefficient $\alpha$, at a given frequency $f$, is:

$$\sigma_a(f) = \frac{7.52k\sqrt{N + N'}}{\sqrt{nZ}\sqrt{NN'}} \text{ (dB/cm)}$$

(2)

Terms used in Eq. (2) are illustrated in figure 1. Here $N$ and $N'$ are the number of independent acoustical lines over which echo data are acquired and analyzed from the reference and the sample respectively; $Z$ is the length of the line segment over which the least squares analysis is applied; $n$ is the number of independent estimates of $ri(f,z)$ over the interval $Z$; and the
factor $k$ is the inverse of the ‘signal to noise ratio’ that is, the mean of the signal intensity to its standard deviation. This ratio is 1 if Rayleigh statistics apply.\textsuperscript{29} Since $n$ is proportional to $Z$, the uncertainty is inversely proportional to the $3/2$ power of the length of the data segments.

Assume the attenuation in the sample is linearly proportional to the frequency, which is approximately true in tissue for frequencies in the range of 1 to 10 MHz. Thus,

$$\alpha_i(f) = \beta_i f$$  \hspace{1cm} (3)

We will seek the attenuation coefficient slope, $\beta_i$ (dB/cm/MHz) as an attenuation parameter throughout this paper. From Eq. (3), $\sigma_{\beta_i}$, the standard deviation of the attenuation coefficient slope at a certain frequency $f_i$ is:

$$\sigma_{\beta_i} = \frac{1}{f_i} \sigma_a(f_i)$$  \hspace{1cm} (4)

One means to decrease the uncertainty in attenuation coefficient estimations without excessively expanding the region over which the estimation is done is to apply spatial angular compounding. Spatial angular compounding during echo data acquisition has proven to be an effective technique for reducing speckle noise to improve B-mode image quality.\textsuperscript{30} However, angular compounding has not yet been used for quantitative ultrasound imaging in the pulse echo mode. An angular compounding scheme described below will be applied for reducing statistical fluctuations.

Another means to decrease errors in attenuation coefficient estimations is to average attenuation coefficients derived from different frequency components of the echo signal spectrum. This technique called ‘frequency compounding’ has been previously applied to reduce speckle in ultrasound images.\textsuperscript{31, 32}

The statistical uncertainty of an attenuation coefficient slope estimation when both spatial and frequency compounding are applied can be described using the following expression (see Appendix):

$$\sigma_{\beta}^\prime = \frac{1}{\sqrt{N_c} \cdot N_f} \cdot \sqrt{\sum_{j=1}^{2N_f} \frac{1}{f_i} \sigma_a^2(f_i)}$$  \hspace{1cm} (5)

where $\sigma_{\beta}^\prime$ is the standard deviation of the attenuation coefficient vs. frequency slope after spatial and frequency compounding, $\sigma_a(f_i)$ is the standard deviation of the attenuation coefficient at a frequency $f_i$ before compounding, $N_i$ is the effective number\textsuperscript{33} of independent signals used for spatial angular compounding, $N_f$ is the effective number of independent frequency components used and $n_f$ is the number of partially-correlated frequency components used. It is readily seen from Eq. (5) that as $N_i$ and $f_i$ increase, $\sigma_{\beta}^\prime$ decreases. Also, it is generally true that as $N_i$ increases, $\sigma_{\beta}^\prime$ drops. The frequency dependence of $\sigma_a(f_i)$ reflects the fact that when attenuation is computed for a small region (see next section), $N_i$ and $N_f$ in Eq. (2), the number of independent beam lines available varies with frequency because echoes from closely spaced beam lines are less correlated at higher frequencies.

**EXPERIMENTAL PROCEDURES**

Initial work was directed towards viewing tradeoffs between resolution and noise during attenuation estimations with compounding. A uniform phantom having an attenuation coeffi-
fi cient slope of 0.5 dB/cm/MHz was scanned. The phantom consists of a water based gel (agarose) with graphite powder uniformly dispersed to provide the needed attenuation. The graphite also serves as a source of backscattered echoes. Because we were interested only in the statistical properties of our estimates, the same phantom was used both as a sample and a reference by scanning different regions to assure independent data.

In addition, a special-purpose phantom was constructed (Fig. 2) to assess the accuracy of the method for determination of local attenuation values, where the above uniform phantom served as the reference and this special-purpose phantom was designated as the sample. The tissue-mimicking materials are solids made rigid through the presence of agarose. In the absence of added scatterers, the backscatter level of these materials is negligible, while the attenuation and propagation speeds are in the range of soft tissue values. Attenuation increases with concentration of bovine milk solids in the material. Milk was concentrated via reverse osmosis at Diehl, Inc., Defiance, Ohio, USA. Backscatter is provided through the presence of various concentrations of glass beads with a mean diameter of about 20 µm (Type 3000E, Potters Industries, Parsippany, NY, USA). The presence of the beads contributes little to the attenuation.

As shown in the diagram in figure 2, there are four parallel cylindrical inclusions in the phantom in which the attenuation coefficient is greater than the surrounding. Cylinders A and D are 1 cm in diameter and are intended for attenuation studies using high frequency transducers. Cylinders B and C are 3 cm in diameter and are for evaluations with low frequency probes. The backscatter coefficient of the material forming cylinders A and B is the same as that in the background material (at any frequency). In cylinder C, the backscatter coefficient is higher than the background value, and in cylinder D it is lower.

During the manufacturing process of each component of the phantom, a 2.5 cm thick, 7.6 cm diameter test cylinder was also made for measurements of propagation speed and attenuation coefficients. Measurements were made using a commonly-used through-transmission method. Values of propagation speed and attenuation coefficients for the various materials

**FIG. 2** Schematic of the attenuation contrast phantom. The targets are cylinders whose diameters are 1 cm (A and D) or 3 cm (B and C). Two scanning windows enable the objects to be imaged at different depths.
at 22°C are shown in table 1. Also shown in this table are the backscatter coefficients relative
to that in the background, measured using apparatus that incorporates a reference phantom.35

Echo data were acquired from each phantom using an Aloka SSD-2000 scanner (Aloka,
Wallingford, CT) equipped with a 3.5 MHz phased array transducer. Because only a low
frequency transducer was available for these studies, we addressed only the larger cylinders
B and C in the inclusion phantom acquisitions. The scan format of this system consists of
120 acoustical lines spread uniformly over a 90° sector. Signals were acquired by tapping
them from a test point in the receiver of the scanner after TGC, but prior to nonlinear process-
ing. Rf echo data were recorded to 12-bit precision at a sampling rate of 50 MHz using a data
acquisition board (CompuScope 12100, Gage Applied Sciences, Inc., Lachines, QUE, Can-
ada) housed in a PC workstation.

To achieve the effects of spatial compounding as would be obtained using a linear array
transducer, the transducer was translated in 0.5 mm increments (Fig. 3) in a direction parallel

---

**TABLE 1** Acoustical properties of the background and the cylindrical target materials in the attenuation phantom.

<table>
<thead>
<tr>
<th>Material</th>
<th>Speed (m/s) (±1 m/s)</th>
<th>Attenuation coefficient (dB/cm) (±0.1 dB/cm)</th>
<th>Slope of attenuation coefficient (dB/cm/MHz)</th>
<th>Relative BSC (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.5 MHz 4.5 MHz 6.2 MHz 8.0 MHz</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Background</td>
<td>1536 1.14 2.08 3.06 3.97 0.48</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cylinder A</td>
<td>1547 1.86 3.55 5.04 6.47 0.79 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cylinder B</td>
<td>1547 1.89 3.57 5.12 6.63 0.80 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cylinder C</td>
<td>1546 1.91 3.54 5.07 6.61 0.80 +3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cylinder D</td>
<td>1547 1.89 3.57 5.12 6.45 0.80 −3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**FIG. 3** Setup to acquire echo data for spatial compounding. The phased array transducer is translated parallel to
the image plane and rf data are acquired every ½ mm.
to the ultrasound image plane, stopping after each translation step to digitize echo signals. For the uniform phantom experiment, a total of 80 such acquisitions were used for both the reference and the sample accordingly, covering a 3.95 cm linear range. For the inclusion phantom experiment, 100 acquisitions covering a 4.95 cm linear range were taken for both the sample and the reference.

Offline analysis consisted first of regrouping the rf echo data to form ‘angled rf data sets,’ as diagrammed in figure 4, each set consisting of data from a specific beam direction but with successive beam lines separated laterally by 0.5 mm. 120 such angled rf data sets were generated, each covering an area of $12.6 \times 3.95 \times \cos(\theta)$ cm for the uniform phantom experiment and $12.6 \times 4.95 \times \cos(\theta)$ cm in the inclusion phantom experiment. The angle $\theta$ is that between the beam line direction of an angled rf data set and the perpendicular.

In the uniform phantom experiment, the reference phantom method was then applied to the echo data within localized data blocks of each angled rf data set. First, echo signal power spectra were computed by applying a sliding 200 point (3 mm) Hanning window and taking a FFT. The windows were allowed to overlap by 50%, providing an optimal compromise between variance reduction in spectrum calculations using the fast Fourier transform and computation time. This calculation was done both for the sample and reference data of each angled rf data set. For the uniform phantom, the matrices of Fourier spectrum values were then each divided into equal sized, nonoverlapping axial segments along the beam direction. Different segment sizes were studied, ranging from 0.5 cm to 2.5 cm. In the cross-beam (lateral) direction, spectral data for neighboring beam lines were grouped, where the lateral extent of a group ranged from 0.25 cm to 1 cm. Thus, nonoverlapping calculation blocks, whose axial extent varied from 0.5 to 2.5 cm and lateral extent from 0.25 to 1 cm were formed (Fig. 5).

Fourier spectrum values at the same depth were averaged within these groups of lines for the sample. Because each angled rf data set was obtained using the same beam line, reference spectra were formed by averaging the Fourier spectra at each depth to decrease the variability of the reference data (Fig. 5). Then the difference between the sample and reference spectra from the same depth was calculated and plotted versus depth over the axial segment of each block. The slope (dB/cm) of the line-fit of these spectral difference values at a specific frequency was derived. Each slope divided by frequency gives the attenuation coefficient slope (dB/cm/MHz) for the block.

The calculation was repeated for select frequencies over the bandwidth of the echo signal. Then frequency compounding was applied by computing the average attenuation coefficient vs. frequency slope among the different frequencies. This final step yielded ‘angled attenuation images.’

Angled images were scan converted to a common reference frame using a registration algorithm that utilized the geometrical relationship between angled rf data sets and the reference frame. Spatial compounding was then used to average attenuation coefficient slopes.
computed for different angles but at the same spatial location. For the inclusion phantom experiment, similar processing was done except that an 80% axial overlap and a 75% lateral overlap were applied to the calculation blocks, which enhances the smoothness of the attenuation image.

**RESULTS**

**A. Uniform phantom results**

After attenuation estimates were derived for all angled data sets in the uniform phantom and scan conversion was done to convert to a rectangular matrix, the mean and standard deviation of local attenuation values were calculated for different block sizes. The standard deviation of the attenuation estimates can be regarded as one criterion that determines the statistical accuracy of the estimation. Standard deviations vs. axial extent of the calculation block are presented in figure 6 for different degrees of spatial compounding and different block widths. Figure 6a presents results for a 1 cm wide calculation block, with the axial size ranging from 0.5 cm to 2.5 cm. Separate data points are used for no spatial compounding ($n_c = 1$), 7 angled data sets compounded ($n_c = 7$) and 15 angled data sets compounded ($n_c = 15$). Figure 6b present similar results, but for a 1 cm long calculation block, with the lateral size ranging from 0.25 cm to 1 cm. In our nomenclature, $n_c$ is differentiated from $N_c$ (Eq. 5) in that $N_c$ is the effective number of independent angles used in spatial compounding while $n_c$ may contain partially-correlated data. Data from neighboring beam angles may not be independent, especially because the V-3.5 transducer has an angular separation between acquisition lines of only 0.75°. Therefore, we included an example where every sixth angled data set separated by $0.75 \times 5 = 3.75^\circ$ was used in compounding, while the number of angles used was still 15 ($n_c' = 15$). Only the center frequency 3.5 MHz (no frequency compounding) was used in these initial spatial compounding cases.
Each example in figure 6 shows that for a given block size, as the number of angles used in spatial compounding increases, the standard deviation drops. More improvement is seen using 15 angled data sets composed of every sixth angled set ($n_c' = 15$) than 15 sets separated
by only 0.75° ($n_c = 15$). Thus, a spatial compounding scheme with more independent data will result in statistically more accurate local attenuation estimations.

The effect that different frequency compounding schemes has on the local attenuation estimations was evaluated in a similar way. Only one angled data set was used (no spatial compounding) while the number of frequencies compounded was varied from $n_f = 1$ (3.5 MHz) – no frequency compounding to $n_f = 3$ (3, 3.5 and 4 MHz) to $n_f = 5$ (3, 3.25, 3.5, 3.75, and 4 MHz) and finally to $n_f' = 5$ (2.5, 3, 3.5, 4, and 4.5 MHz). The results are shown in figure 7. Figure 7a presents results for a 1 cm wide calculation block of axial sizes ranging from 0.5 cm to 2.5 cm; figure 7b presents similar results, but for a 1 cm long calculation block of different lateral sizes, ranging from 0.25 cm to 1 cm. As the number of frequencies used in frequency compounding increases, the standard deviation drops for the same block size. Similar to the spatial compounding case, $n_f$ is differentiated from $N_c$ (Eq. (5)) in that $N_c$ is the effective number of independent frequency components, while $n_f$ is the number of partially correlated frequency components used. Comparison of results for $n_f' = 5$ with those for $n_f = 5$ illustrates that a frequency compounding scheme with more independent data will result in a smaller standard deviation of local attenuation estimations. The mean attenuation difference for the sample and the reference measured in all the above cases is close to zero, which is expected since the sample and the reference are from the same uniform phantom.

### B. Attenuation phantom results

For the attenuation images of the inclusion phantom, a combined compounding scheme in which fifteen angles with $0.75 \times 5 = 3.75°$ angular separation ($n_f = 15$) and 5 frequency components – 2.5, 3, 3.5, 4, and 4.5 MHz ($n_f' = 5$) was utilized. The calculation block used was 1 cm axially by 1 cm laterally, with an overlap of 80% in the axial direction and 75% in the lateral direction. This scheme should result in a standard deviation of the attenuation coefficient slope of approximately 0.07–0.1 dB/cm/MHz, which can be predicted as follows.

For a single acquisition angle, the standard deviation with frequency compounding (Eq. (5)) for this scheme can be estimated to be 0.2 dB/cm/MHz. This value is obtained by looking at the 1 cm data point on the $n_f' = 5$ curve in figure 7a. For spatial compounding, each cylinder was centered at a depth of 5.5 cm during the scan and has a diameter of 3 cm. Figure 8 illustrates the number of angled data sets available throughout the imaged field for this set up; in the area of the inclusions, there are between $n_f' = 8$ and $n_f' = 15$. Thus, the minimum value of $N_{c_{\text{MAX}}}$ (in Eq. (5)) can be estimated by taking the ratio of $\sigma_{\theta \theta}^j$ for the case of $n_f' = 15$ to $\sigma_{\theta \theta}^j$ for $n_f' = 1$ (no compounding) for the 1 cm axial block size in figure 6a. This yields $N_{c_{\text{MIN}}}^{-1/2} \approx 0.357$, which means $N_{c_{\text{MAX}} \approx 7.8}$. If we assume the number of independent estimations is proportional to the number of partially-correlated estimations, then, $N_{c_{\text{MIN}}} = 7.8 \times 15/8 \approx 4.2$, and the maximum value of $N_c^{-1/2}$ is roughly 0.488.

Substituting these extremes into Eq. (5) along with the standard deviation for frequency compounding, we get $0.07 \text{ dB/cm/MHz} \leq \sigma_{\theta \theta}^j \leq 0.1 \text{ dB/cm/MHz}$. Since the contrast between cylinders and background is roughly 0.3 dB/cm/MHz, the contrast-to-noise ratio is expected to be between 3 and 4.5 in the inclusion area.

Attenuation images that include cylinders B and C in the attenuation phantom are presented in figure 9. From the attenuation image data, we measured mean attenuation values to be 0.73 dB/cm/MHz for cylinder B and 0.70 dB/cm/MHz for cylinder C. The background region image data are at 0.42 dB/cm/MHz. These data should be compared with the direct measurements of values using test cylinders in a water tank (Table 1). Also, we can see the improvement gained with compounding by comparing figures 9b and 9d with their noncompounded counterparts figures 9a and 9c. The contrast-to-noise ratio in the inclusion areas of
Attenuation imaging using compounding

Figure 9 are 4.2 for inclusion B and 3.4 for inclusion C. Both values are in the expected range, as detailed in the previous paragraph.

**Figure 7** Standard deviation of attenuation determinations in the uniform phantom vs. axial and lateral extent of the calculation block for different degrees of frequency compounding. As in figure 6, \( n_f \) is the number of frequencies from which attenuation values were compounded. Figure 7a presents similar results for a 1 cm wide calculation block of axial sizes ranging from 0.5 cm to 2.5 cm; figure 7b presents similar results, but for a 1 cm long calculation block of different lateral sizes, ranging from 0.25 cm to 1 cm.
DISCUSSION

This paper shows that by using compound data acquisition and analysis with a reference phantom, it is possible to measure attenuation coefficients in small, localized blocks and derive coarse attenuation images. Using a manually-translated 3.5 MHz phased array transducer to simulate compound acquisition, 1 cm $\times$ 1 cm calculation blocks (20 acoustic lines) yielded accurate attenuation images of a phantom containing 3 cm diameter inclusions. The standard deviation of the attenuation coefficient vs. frequency slope was less than 0.1 dB/cm/MHz for 1 cm $\times$ 1 cm calculation blocks.

In a similar study, He$^{37}$ reported a standard deviation of attenuation coefficient vs. frequency to be about 0.2 dB/cm/MHz for a calculation block size of 2 cm by 350 A lines, covering all available cross section of a phantom with a diameter of 7.5 cm. Walach$^{26}$ estimated the standard deviation to be about 0.14 dB/cm/MHz for a 1.6 cm $\times$ 0.4 cm calculation block. Neither of these studies applied spatial compounding. The calculation block used in this paper is still relatively large, which puts a limit on the resolution of present attenuation images. As a result, smaller, 1 cm diameter cylinders in the attenuation phantom were not imaged with this implementation, although they likely will be imaged when this method is applied using higher frequency transducers.

FIG. 8 Image whose gray scale value at each location indicates the number of angled data sets available for compounding for the data acquisition applied to the attenuation contrast phantom.
FIG. 9 Attenuation images of regions in the attenuation phantom which include the 3 cm diameter cylindrical targets whose attenuation is 0.3 dB/cm/MHz greater than that of the background. Panels (a) and (b) are noncompound attenuation image and attenuation image with both spatial and frequency compounding of cylinder B, whose backscatter is the same as that of the background. Panels (c) and (d) are noncompound attenuation image and attenuation image with both spatial and frequency compounding of cylinder C, whose backscatter is 3 dB greater than the background. The units of the color-bar are dB/cm/MHz.
In this work, we assumed a linear relationship between the attenuation coefficient and frequency. In fact, a nonlinear dependence of attenuation on frequency, such as $\alpha = B_s f^q$, can also be accommodated by a least-squares fit of attenuation coefficients at different frequencies, where both $B_s$ and the power term $q$ can be derived from experimental data. The error propagation for $B_s$ would be analogous to that used for Eq. (5) and the error propagation for the power term $q$ could be derived as well.

Eqs. (2) and (5) provide useful guidance on the degree of statistical accuracy that can be obtained using compounding. Obviously, increasing the number of independent data sets that can be applied to any given area improves the results. This paper has emphasized both spatial and frequency compounding to increase $N_c$ and $N_f$. Other methods that will be applied in future research include increasing the slice thickness and spatial compounding from points outside the image plane.

The current algorithm is sensitive to changes in backscatter. An increase in echogenicity will be viewed as a negative change in the attenuation and vice versa, which can be seen in the attenuation image of cylinder C (Fig. 9d). Here, an increase in backscatter at the top boundary of the cylinder is interpreted by the algorithm as a sharp decrease in attenuation. We are studying whether this limitation can be overcome using an adaptive attenuation filter that would omit abrupt signal amplitude changes. Admittedly, the work was done for relatively uniform samples and any increase in the homogeneity of the imaged region will add to image fluctuations.

Clinically, the appearance of ‘posterior echoes,’ either ‘enhancement’ or ‘shadowing,’ is the fundamental criteria for assessing attenuation within focal lesions such as the breast, kidney or thyroid. The attenuation coefficient slope has been identified by d’Astous and Foster$^{38}$ and Landini et al$^{39}$ as a discriminating parameter for breast tissue. Using a 3.5 MHz transducer, the error of local attenuation estimates within a 3 cm mass in a phantom was predicted to be less than 10%. Although the masses studied here are homogeneous, the technique shows promise for providing methods to quantify attenuation within breast masses, particularly for scans derived at ultrasound frequencies above 7 MHz. Our vision is that a clinical user will draw a region of interest within the suspected mass on an ultrasound breast image. The algorithm then will retrieve all rf data associated with that region and compute the attenuation. Similarly, the relative difficulty of beam penetration is the current criterion for estimating ultrasound attenuation in large organs such as the liver. The attenuation estimation method described in this paper could readily be incorporated into modern digital scanners and its role assessed for aiding these subjective assessments. Maklad et al$^{40}$ in an in vivo study, reported attenuation values in normal livers of $0.52 \pm 0.03$ dB/cm/MHz, while in alcoholic cirrhosis, they were estimated as $0.83 \pm 0.09$ dB/cm/MHz. Similarly, Lu$^{10}$ reported values for normal liver to be $0.55 \pm 0.07$ dB/cm/MHz and values of $0.85 \pm 0.08$ dB/cm/MHz for fatty infiltrated liver. Thus, in large organs, if the attenuation coefficient depends on the degree of infiltration, the compound schemes described here should enable estimation of degree of steatosis easily. Furthermore, in cases where the liver exhibits inhomogeneous attenuation caused by focal fatty infiltration, the ability to quantify attenuation over local regions on the order of 2-3 cm should add value to clinical use of this data and assist in differentiation of this condition.

CONCLUSION

A method for measuring ultrasound attenuation, which uses both spatial and frequency compounding in data acquisition and analysis, has been described. The technique applies a reference phantom to account for imaging system dependencies of echo signals and computes attenuation over segments of 0.5 cm to 2.5 cm in length. Results demonstrate an effec-
Attenuation reduction of the variance in attenuation measurements when compounding is applied vs. when no compounding is used. The technique is useful for measuring attenuation over small regions on ultrasound images and for constructing coarse attenuation images.

ACKNOWLEDGEMENTS

The authors thank Gary Frank, Anthony Gerig and Udomchai Techavipoo for their technical assistance. This work was supported in part by NIH grants R21EB00722, R01CA39224 and R42GM54377.

APPENDIX

Let $N_f$ be the effective number of independent signals used for spatial angular compounding, $N_f$ the effective number of independent frequency components used in frequency compounding, and $n_f$ the number of partially-correlated frequency components used.

Using frequency compounding, the average slope of attenuation coefficient vs. frequency, $\beta'$ (dB/cm/MHz) is obtained from $\beta_i$, the attenuation coefficient slope at each measurement frequency $f_i$. That is,

$$\beta' = \frac{\sum_{i=1}^{n_f} \beta_i}{n_f}$$

So the standard deviation of $\beta'$, $\sigma_{\beta'}$ is:

$$\sigma_{\beta'} = \sqrt{\frac{\sum_{i=1}^{n_f} \sum_{j=1}^{n_f} \text{cov}(\beta_i, \beta_j)}{n_f}} = \sqrt{\frac{\sum_{i=1}^{n_f} \sigma_{\beta_i}^2 + \sum_{i=1}^{n_f} \sum_{j=1}^{n_f} \text{cov}(\beta_i, \beta_j)}}{n_f}$$

Let

$$N_f = \sqrt{\frac{n_f}{\sum_{i=1}^{n_f} \text{cov}(\beta_i, \beta_j)} + \frac{\sum_{i=1}^{n_f} \sigma_{\beta_i}^2}{1 + \sum_{i=1}^{n_f} \sigma_{\beta_i}^2 \rho(\beta_i, \beta_j)}}$$

$$\sigma_{\beta'} = \frac{1}{N_f} \sqrt{\frac{\sum_{i=1}^{n_f} \sigma_{\beta_i}^2}{1 + \sum_{i=1}^{n_f} \sigma_{\beta_i}^2 \rho(\beta_i, \beta_j)}}$$

where $\sigma_{\beta_i}^2$ is the variance of the estimate at frequency $f_i$, $\rho(\beta_i, \beta_j)$ is the correlation of attenuation coefficient slope, estimated at frequency $f_i$ and $f_j$. 


Combining with Eq. (4),

\[ \sigma_{\beta}^2 = \frac{1}{N_f} \left( \frac{1}{f_i^2} \right) \sum_{i=1}^{n} \sigma_n^2(f_i) \]  \hspace{1cm} (A.5)

After spatial compounding,

\[ \sigma_{\beta}^2 = \frac{1}{N_f} \cdot \frac{1}{N_f} \cdot \left( \frac{1}{f_i^2} \right) \sum_{i=1}^{n} \sigma_n^2(f_i) \]  \hspace{1cm} (A.6)

\section*{REFERENCES}


