Performance Optimization in Elastography: Multicompression with Temporal Stretching

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A general theoretical framework known as the strain filter has been previously used to evaluate the performance in elastography. The strain filter describes the relationship among the resolution, dynamic range, sensitivity and elastographic SNR (SNRe), and may be plotted as a graph of the upper bound of the SNRe vs. the strain experienced by the tissue, for a desired elastographic axial resolution as determined by the data window length. The ideal strain filter has an infinitely high, flat all-pass characteristic shape in the strain domain, which means that all local tissue strains are displayed in the elastogram with infinite SNRe; it also means that the strain dynamic range in the elastogram is infinite as well. Practical strain filters obtained using a single tissue compression have a bandpass characteristic shape in the strain domain, where the −3 dB width of this bandpass characteristic may be defined as the elastographic dynamic range. In this paper, we present an optimal technique for stretching multicompression elastography, practiced by selecting the optimum incremental applied strain using the strain filter. Two techniques, temporal stretching and multicompression elastography, are combined in this paper to improve elastogram quality. Stretching multicompression elastography using the optimal applied strain increment alters the shape of the strain filter from its bandpass characteristic to a more desirable high-impedance filter. The dynamic range of optimal stretching multicompression elastography is limited only by tissue nonlinearities. This optimal applied strain increment minimizes signal decorrelation and achieves the maximum achievable elastographic SNRe.

KEY WORDS: Dynamic range; elastography; multicompression; strain filter; temporal stretching; ultrasound. ©1996 Academic Press, Inc.

I INTRODUCTION

Traditionally, elastography employed a single compression and time-delay estimation between the pre- and the post-compression rf signal to estimate tissue strain [1,2]. However, signal decorrelation, a significant source of error in the strain estimate, limits the range of tissue strains that can be accurately imaged. Techniques to minimize decorrelation errors are therefore essential to improve elastogram SNRe. The elastographic SNRe is defined in terms of the mean-to-standard deviation ratio in the elastogram [3,4]. Two techniques, temporal stretching [4–7] of the postcompression signal to align the rf peaks with the precompression signal and multicompression averaging [6–9], which reduces signal decorrelation by using small compressions, have been employed in the past to reduce signal decorrelation. Multicompression averaging utilizing small compressions reduces decorrelation artifacts in all three dimensions, while temporal stretching further reduces axial signal decorrelation. The ability of temporal stretching to reduce signal...
decorrelation errors deteriorates rapidly with increasing strain. In addition, decorrelation errors due to lateral and elevational displacement of the scatterer also increase.

The combination of the two techniques above produces marked improvements in the elastogram. Multicompression averaging [6–9] in conjunction with temporal stretching [4–7] of the postcompression rf signal has been used previously in elastography to improve the sensitivity, dynamic range and the signal-to-noise ratio in the elastogram (SNR_e) by reducing signal decorrelation and electronic noise artifacts [6,7]. Stretching multicompression elastography [6,7] provides a method of obtaining elastograms where the compressive force is applied in stages, with a small compressive force used in each stage. Small compressions along with temporal stretching minimize errors due to signal decorrelation and significantly improve the elastogram SNR_e.

A theoretical framework to characterize the quality of elastograms for a single compression was developed using the strain filter approach by Varghese and Ophir [10], which incorporates the effects of signal decorrelation. The strain filter quantifies the elastographic image quality (sensitivity, dynamic range and elastogram SNR_e) at a fixed value of the resolution, given the signal processing and system parameters [10] (also discussed in Appendix B). The performance of the strain estimator is predicted assuming ideal tissue compressibility conditions. This assumption generally results in significant mechanical artifacts in the elastogram. These artifacts can be reduced by solving the inverse problem [11–15]. This model does not account for the possible hardening of soft tissue with applied strain or other tissue nonlinearities. In this paper, we present an optimal technique for stretching multicompression elastography, practiced by selecting the optimum applied strain increment using the strain filter.

Multiple displacement estimates from successive small compressions have been accumulated by O’Donnell et al. [8,9] to improve the quality of their elasticity images. Their phase sensitive speckle tracking algorithm is able to detect only small differential displacements (<λ/4 per pixel, where λ is the wavelength of the ultrasound pulse) without ambiguity. Averaging multiple displacement estimates reduces the variance by a factor of N (where N is the number of estimates), thereby increasing the SNR_e in the image by a factor of √N (assuming uncorrelated strain estimates, see Appendix D for details), as reported in [9]. However, they do not temporally stretch the postcompression signal before strain estimation [3,4]. Elastography [1,2] is not limited by the λ/4 single applied strain increment in [8,9].

This paper demonstrates that an optimum applied strain increment exists for multicompression elastography. This is demonstrated by using the strain filter approach. The next section discusses the optimization procedure and criteria for optimization. Simulations to validate the theoretical model are presented in section III. The contributions of this paper are discussed and summarized in section IV.

II THEORETICAL OPTIMIZATION OF THE STRAIN FILTER

A complete characterization of the performance of the strain estimator in elastography is obtained using the strain filter [10] (discussed in Appendix B). The strain filter incorporates the effects of signal decorrelation, quantization and electronic noise in the estimation model. Signal decorrelation is modeled as a noise process that reduces the composite
signal-to-noise ratio (SNR<sub>C</sub>) in the echo signal (refer to Appendix B). The strain filter concept discussed by Varghese and Ophir [10] is extended in this paper to model the optimum stretching-multicompression elastography technique.

Optimization of the performance of the strain estimator in elastography is discussed in terms of increasing the area under the strain filter curve. Enhancement of the area under the strain filter curve implies an increase in the SNR<sub>e</sub><sup>UB</sup> (upper bound of the SNR<sub>e</sub>) or the strain dynamic range or both for the same elastographic resolution. Increasing the area under the strain filter curve therefore improves the elastogram quality.

Both temporal stretching and multicompression elastography improve the elastogram quality. A combination of these two techniques, however, produce a marked improvement in the elastograms, as discussed by Varghese et al. [7]. In this section, we discuss the improvement in elastogram quality obtained by each technique separately and by a combination of both. We demonstrate that the optimal improvement in elastogram quality (maximum area under the strain filter curve) is obtained using multicompression elastography in conjunction with temporal stretching, practiced by selecting the optimal applied strain increment. The discussion in this paper applies only for uniformly elastic tissue media, where the applied strain is the same as the tissue strain.

(a) Temporal Stretching of the Postcompression Echo Signal

A theoretical strain filter curve obtained before and after temporal stretching for a single tissue compression is presented in figure 1. The following are typical signal parameters used to compute the strain filter [10]: \( T = 1 \text{ mm} \) (1.3 \( \mu \text{s} \)), \( f_0 = 5 \text{ MHz} \), \( B = 3 \text{ MHz} \) (rectangular bandwidth), SNR<sub>s</sub> = 100 (40 dB), and interval between strain estimates \( \Delta x = 0.5 \text{ mm} \) (\( \Delta t = 0.66 \mu \text{s} \)).

Signal decorrelation errors increase with tissue strain, reducing the value of the correlation coefficient, and causing \( \sigma_{ZZLB}^2 \) (the Ziv-Zakai lower bound on the variance, refer to Appendix B for details) to move from the Cramér-Rao Lower Bound (CRLB) to the Barankin bound or the constant variance level as shown in Eq. (B-5). The three distinct regions in Eq. (B-5) are observed in plots of the strain filter in figure 1. The three distinct regions in the strain filter obtained after temporal stretching are the CRLB for strains <20\%, Barankin bound for strains >20\% and <30\%, and the constant variance level for strains >30\%. For the strain filter without temporal stretching, the CRLB region is limited to strains <10\%. As the lower bound coincides with the Barankin bound, SNR<sub>e</sub><sup>UB</sup> drops sharply, with a further drop in the SNR<sub>e</sub><sup>UB</sup> observed as the variance coincides with the constant variance level.

A significant increase in the area under the strain filter with temporal stretching is observed when compared to the case where stretching is not performed. Note also the enhancement in SNR<sub>e</sub><sup>UB</sup>, sensitivity, and dynamic range obtained using temporal stretching in figure 1. Temporal stretching therefore significantly improves the elastogram quality by reducing axial signal decorrelation (since we can correct only for axial motion of the tissue scatterers). If only temporal stretching were used to optimize the elastogram, a tissue compression with maximum SNR<sub>e</sub><sup>UB</sup> is the ideal choice (near the top of the strain filter).

However, practical implementation of temporal stretching has been successful only for small tissue strains. Note also the progressive flattening of the strain filter curve in figure
FIG. 1 The theoretical strain filters obtained using a single tissue compression incorporating temporal stretching (σσσσ) and without temporal stretching (****). Note the enhancement in the SNR_{UB}^e sensitivity and dynamic range obtained using temporal stretching without sacrificing resolution.

1 (with temporal stretching), which demonstrates the reduction in the rate of increase in SNR_{e} with increased compression. This progressive flattening of the strain filter curve is due to the inability of the temporal stretching operation to completely reverse the effects of signal decorrelation. Improvements in elastogram quality obtained using temporal stretching thereby reduce with an increase in the tissue strain.

(b) Optimization of Multicompression Elastography

Multicompression elastography (see Appendix D for details), performs strain imaging over a number, N, of small applied strain increments, s_{s}, which minimizes signal decorrelation in all three directions of tissue motion. The use of a small applied strain increment with multicompression elastography minimizes the effects of signal decorrelation in all three dimensions. Since the applied strain is small, the motion of the tissue scatterers is limited, thereby minimizing signal decorrelation. The applied strain increment determines the optimum SNR_{UB}^e value obtained using multicompression elastography. The expression used for the graphical optimization of the SNR_{UB}^e in this paper is given in Appendix D (Eq. (D-1)).

This optimal applied strain increment is obtained by plotting SNR_{UB}^e versus the number of compressions (N), using the strain filter without temporal stretching, as shown in figure
FIG. 2 SNR\textsuperscript{UB} obtained using the strain filter without temporal stretching plotted as a function of the number of compressions (N) for a 1% (×××) and 2% (○○○) total tissue strain. The peak value of SNR\textsuperscript{UB} occurs at N = 3 and 5 for a total tissue strain of 1% and 2% respectively (optimal compression step of 0.33% and 0.4% tissue strain).

2. The peak or optimum performance is obtained in each case for an incremental applied strain of \(\approx 0.33\%\) (i.e., the peak occurs for N = 3 and 5 for the total tissue strain of 1% and 2%, leading to optimal applied strain increments of 0.33% and 0.4%, respectively, as seen in figure 2). The smaller applied strain increment is chosen, since the rate at which SNR\textsuperscript{UB} decreases is smaller for small applied strain increments, as observed from figure 2. In addition, the effects of signal decorrelation are less pronounced with the smaller applied strain increment.

The strain filter obtained using this optimal applied strain increment for multicompression elastography is presented in figure 3, along with the strain filter obtained using single compression before and after temporal stretching. The total applied strain, or total stroke applied to the tissue, is the same for all the techniques shown in figure 3. The behavior of the strain filter changes from its bandpass characteristic shown in figure 1 to a high-emphasis filter using the multicompression technique. The dynamic range of the strain filter is therefore limited only by the maximum compressive force that can be safely applied to tissue [16]. To calculate the area under the strain filter (for optimization purposes), we impose a hard limit on the total strain applied to the tissue at 20%. This value was chosen from previous research studying the onset of tissue stress-strain nonlinearities in compression studies on raw meat [17]. In the presence of such tissue nonlinearities, the performance predicted by the strain filter analysis is no longer applicable.
FIG. 3 Optimal strain filter obtained using multicompression averaging (x-x-x) along with the single compression strain filters with temporal stretching (o-o-o) and without temporal stretching (x*x*x). The optimal applied strain increment of 0.33% was obtained from figure 2. Note the change in the strain filter from a bandpass to high-emphasis filter with multicompression averaging.

Note from figure 3, that although the area under the strain filter curve obtained using optimal multicompression elastography is larger than that obtained from single compression without the application of temporal stretching, the strain filter curve with temporal stretching encloses a larger area (and is therefore a more optimal technique). In addition, SNR_{UB} is significantly higher than the optimal multicompression strain filter, especially for low tissue strains.

The strain filter with temporal stretching, however, is limited by signal decorrelation effects, which arise with increased tissue compression. Multicompression techniques, due to the small applied strain increment, is not limited by signal decorrelation. Since both the techniques mentioned above can be applied in conjunction with each other, we can potentially obtain the enhancements in elastogram quality produced by both these techniques. We now study the improvement in the elastogram quality obtained using stretching-multicompression elastography. The limitations of both the techniques mentioned above are eliminated as illustrated in the next section.

(c) Optimization of Stretching-Multicompression Elastography

Combining the two techniques mentioned above should generate a strain filter curve that would enclose the single compression strain filter with temporal stretching, and,
in addition, grow at a faster rate with tissue compression when compared to the multicompression technique. Figure 4 plots $\text{SNR}_{e}^{\text{UB}}$ as a function of the number of compressions (N) for different values of the strain using the strain filter with temporal stretching. The optimal performance is obtained in each case for a incremental applied strain of $\approx 0.5\%$ (i.e. the peak occurs for $N = 2$ and 4 for the total tissue strain of $1\%$ and $2\%$, respectively, as seen in figure 2). Observe from figures 2 and 4 that the value of the optimal applied strain increment increases with temporal stretching from $0.33\%$ to $0.5\%$ tissue strain.

The strain filter for the stretching-multicompression technique using the optimal applied strain increment determined above is plotted in figure 5. Note from the figure 5 that this strain filter has the largest area under the curve and therefore generates elastograms with the largest value of the $\text{SNR}_{e}^{\text{UB}}$, dynamic range and sensitivity without sacrificing resolution. Such a filter optimizes the combination of temporal stretching and multicompression averaging and generates the highest quality elastograms. The area under the optimized strain filter is almost doubled when compared to the single compression strain filter with temporal stretching (obtained by using the hard limit at $20\%$ tissue strain in figure 5).
FIG. 5 Optimal strain filters obtained using multicompression averaging (×××) and multicompression averaging with temporal stretching (−−−) along with the single compression strain filter with temporal stretching (○○○). The optimal applied strain increments used for the multicompression strain filters were obtained from figures 2 and 4.

(d) Relationship between the Optimal Applied Strain Increment and the Inflection Point on the Optimized Single Compression Strain Filter

The optimized strain filter for stretching multicompression elastography breaks away from the single compression strain filter curve with temporal stretching at the optimal applied strain increment of 0.5% and increases asymptotically (because of minimal signal decorrelation) as illustrated in figure 6. Note that this particular applied strain increment corresponds to the inflection point for the single compression strain filter. The inflection point corresponds to the tissue compression beyond which temporal stretching is unable to completely compensate for signal decorrelation (due to movement of tissue scatterers in the lateral and elevational directions) along with the subsequent decrease in the rate of increase of $\text{SNR}^\text{UB}_\varepsilon$. This is discussed in detail in the following paragraphs.

Figure 7 presents tissue strain and its standard deviation plotted on a log-log scale. The ratio of the tissue strain to the standard deviation (defined as the strain filter in Appendix B) is now presented as the difference between these two curves. The enclosed region between these two curves defines the strain filter for the single compression case. The difference between these two curves is proportional to the $\text{SNR}^\text{UB}_\varepsilon$ for specified tissue strains. Three regions are observed in the standard deviation curve: the segment (referred to as A), where the standard deviation is constant with increasing strain; segment B, where the standard deviation is proportional to strain, and in the third region (segment C), where
FIG. 6 Strain filters obtained using single compression ($\sigma$) and multicompression elastography with temporal stretching, using the optimal multicompression step of 0.5% (—). The optimum applied strain increment occurs at the inflection point of the single compression strain filter. Note that the optimized multicompression strain filter breaks away from the single compression strain filter at the inflection point (optimal applied strain increment).

standard deviation increases at a higher rate than the tissue strain (Barankin bound and the constant variance level).

In this paper, we discuss only the first two segments (A and B) of the standard deviation curve. In segment A, the standard deviation is constant (bounded by CRLB), and intersects the tissue strain curve. The $\text{SNR}_e^{\text{UB}}$ to the left of the intersection is less than 1, which implies that these tissue strains cannot be accurately measured. The difference between the two curves increases with tissue strain (indicating an improvement in $\text{SNR}_e^{\text{UB}}$) in segment A. However, with a further increase in tissue strain, the difference between the curves becomes constant (segment B), which means that the ratio is a constant (observe the flattening of the strain filter curve in figure 1). The knee of the standard deviation curve (between segments A and B) occurs at the smallest compression that still achieves the maximum value $\text{SNR}_e^{\text{UB}}$ (largest difference between the curves), and also determines the optimum applied strain increment (since the rate at which $\text{SNR}_e^{\text{UB}}$ increases is maximum at this point). The optimal applied strain increment obtained by computing the point on segment B, where the difference between the two curves is maximum for the smallest tissue strain value, is approximately 0.5%.

Figure 8 presents a comparison of the optimized strain filters obtained using single compression and multicompression elastography with temporal stretching. The multicompression curves for applied strain increments of 0.075% and 2% (suboptimal increments)
FIG. 7 Tissue strain (—) and its standard deviation (××××) bounded by the ZZLB) plotted on a log-log scale. The ratio of the tissue strain to the standard deviation (defined as the strain filter) is now the difference between these two curves. The knee of the standard deviation curve (between segments A and B) gives the smallest applied strain that still achieves the maximum value \( \text{SNR}^{\text{UB}} \) (largest difference between the curves), which is the optimum compression increment.

are also shown in figure 8. Note the reduction in the \( \text{SNR}^{\text{UB}} \) particularly for small values of the tissue strain for the cases where the optimal multicompression increment is not employed. A simulation experiment is presented in the next section to validate the theoretical analysis.

III SIMULATION

The performance of the optimal multicompression technique with temporal stretching for obtaining elastograms is tested using simulated rf echo signal pairs. The normalized crosscorrelation function was used to estimate the time-delay (displacement) values.

(a) Method

The pre- and postcompression echo signals are generated using a transducer with a center frequency of 5 MHz and standard deviation of 0.21 \( \mu \)s for the Gaussian envelope (−3 dB bandwidth of 3 MHz). The rf echo signals were sampled at 50 MHz. In all the simulations, crosscorrelation processing was performed using a 1 mm overlapping window
FIG. 8 Strain filters obtained using single compression (o-o-o) and multicompression elastography with temporal stretching, using the optimal multicompression step of 0.5% (—), and suboptimal increments of 0.075% (x-x-x) and 2% (###) respectively.

with 0.5 mm (0.66 μs) overlap between consecutive windows.

The transducer is modeled as a one-dimensional sampled aperture composed of point subtransducer elements equally spaced by λ/2. Each subtransducer element is modeled as a point source or receiver with a two-way Gaussian transfer function. The impulse response of the transducer aperture at a given point in the beam field is computed as the summation of appropriately scaled and delayed subtransducer impulse responses.

The scattering medium is represented by a two-dimensional array of random numbers corresponding to the point scatterer locations. The scatterer density in the media was set to 48 scatterers/pulse width. The elastic target is compressed between two large compressing surfaces to simulate a constant stress distribution. In addition, lateral and elevational decorrelation effects due to scatterer motion are ignored. The compressor applies unidirectional stress to the target such that all scatterers within the ultrasonic beam move in the direction of the applied compression. The tissue stress is assumed to be uniformly distributed so that the localized stress is constant throughout the medium. The displacement of each scatterer is a function of the tissue strain, and is modeled by considering an equivalent one-dimensional spring system described by Céspedes and Ophir [3]. Since this model considers only static compression, viscous and inertial terms are ignored.

A precompression rf echo signal is obtained from the randomly-distributed scatterers by convolving them with the ultrasound pulse. Only scatterers within a 1.5 mm rectangular area around the transducer axes were used in the computation of the rf A-line. Each scatterer location is then changed depending on the compressive force and a post-compression rf
FIG. 9 Simulation results (×××) with temporal stretching using a 0.5% applied strain increment. The theoretical SNR\textsubscript{cUB} values (—) obtained using the strain filter at 0.5% applied strain along with the √N improvement with multicompression averaging are also presented.

echo signal generated. The process is repeated for 28 different lateral locations separated by 3 mm in the simulated phantom to obtain independent rf echo signal pairs. The strain values are computed from the individual rf echo signal pairs.

(b) Results

Figure 9 illustrates the improvement in the SNR\textsubscript{c} obtained using stretching multicompression elastography with an applied strain increment of 0.5%. Each point in figure 9 plots the mean SNR\textsubscript{c} value and its standard deviation (error bars), over 28 independent simulations. The X axis represents the number of compressions, and SNR\textsubscript{c} is plotted along the Y axis. The elastograms obtained during each applied strain increment are averaged to obtain the improvement in the SNR\textsubscript{c}, seen in the figure, with an increase in the number of compressions. The √N improvement in the simulation results is observed from the figure.

Theoretical performance plots using the optimized strain filter for stretching multicompression elastography with the optimal applied strain increment are also plotted in figure 9 along with the simulation results. The theoretical curve indicates the upper bound on the SNR\textsubscript{c} obtained using this algorithm. The theoretical curve shows the √N improvement in the SNR\textsubscript{cUB} (obtained from the strain filter with the 0.5% applied strain increment) with multiple compressions. The simulation results are bounded by the theoretical performance
curve. Note that the simulation results also follow the same trend as the theoretical curve. The close correspondence between the theoretical and simulation results provide a validation of the theoretical strain filter model used in this paper. The one-dimensional mechanical model used in this paper is sufficient for modeling uniformly elastic targets. For nonuniform media, an adaptive implementation of the above technique is employed, which will be discussed in a subsequent publication.

IV DISCUSSIONS AND CONCLUSIONS

The performance enhancement obtained using optimal stretching multicompession elastography is presented in this paper. Optimal multicompession elastography uses the smallest applied strain increment that minimizes signal decorrelation, and thereby provides the maximum achievable $\text{SNR}_e$ in the elastogram. All other applied strain increments (smaller or larger than the optimal increment) are sub-optimal (reduce the area under the strain filter curve) and provide reduced performance (as observed in figure 8). However, it has to be noted that the optimal applied strain increment obtained in this paper is applicable only for uniformly elastic tissue media. In addition, the optimal compression increment is dependent on system parameters. Incorporation of the effects of lateral and elevational signal decorrelation into the strain filter, will also change the optimal compression increment.

In this paper, we discuss the improvement in elastogram quality obtained by using temporal stretching and multicompession separately and a combination of these two techniques to obtain elastograms with the highest quality (largest area under the strain filter curve). Combining the two techniques mentioned above generates a strain filter that encloses the single compression strain filter with temporal stretching, and, in addition, grows at a faster rate with tissue strain when compared to the multicompession technique alone.

The optimized strain filter for stretching multicompession elastography breaks away from the single compression strain filter curve with temporal stretching at the optimal applied strain increment of 0.5%, which also corresponds to its inflection point. The inflection point corresponds to the tissue compression beyond which temporal stretching is unable to completely compensate for axial signal decorrelation. The shape of the strain filter also changes from its bandpass characteristic for the single compression case to a more desirable high emphasis filter using the optimum applied strain increment. This change in the strain filter is attributed to the fact that the variance of the strain estimate is always bounded by the CRLB under these conditions. The bandpass nature of the single compression strain filter is caused by signal decorrelation triggering the rapid increase in the variance of the strain estimator (observe regions B and C in figure 7). With the use of the optimal applied strain increment, the variance of the strain estimator is maintained near the knee (between segments A and B) of the standard deviation curve in figure 7.

Since the strain filter for the optimal multicompession elastography case is a high-emphasis filter, the dynamic range is now limited only by tissue inhomogeneties. Improvements in both the dynamic range and elastographic $\text{SNR}_e$ are obtained using this technique when compared to single compression elastography with temporal stretching at the same
elastographic resolution. Due to the high emphasis nature of the optimized multicompression strain filters (see figure 5), softer regions in tissue (areas that can accommodate larger tissue strains) are seen in the elastogram with a high SNR_\text{e}, when compared to stiffer tissue. However, the optimized multicompression strain filter with temporal stretching affords the best available SNR_\text{e} in both the softer and stiffer tissue areas.

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APPENDIX A

Variance of the Strain Estimator

Axial strain (s) is the displacement gradient, which is estimated from two adjacent time delay estimates separated by an time interval (\Delta t) [1], assuming a constant speed of sound in the tissue, viz

\[ s = \frac{\tau_2 - \tau_1}{\Delta t}, \quad (A-1) \]

where \( \tau_1 \) is the time delay estimate at time \( t \), and \( \tau_2 \) is the time delay estimate at time \( t + \Delta t \).

Since the strain estimate is obtained from a linear combination of two random variables (time delay estimates separated by \( \Delta t \)), the variance of the strain estimator depends on the variance of the time delay estimator. Assuming stationarity, the variance of the strain estimate (\( \sigma^2_s \)) is expressed in terms of the variance of the time delay estimates (\( \sigma^2_\tau \)) in [18], and is given by

\[ \sigma^2_s \geq \frac{2\sigma^2_\tau}{T\Delta t}, \quad (A-2) \]

where \( T \) is the duration of the data segment. Eq. (A-2) illustrates that for a given window size and overlap (\( \Delta t \)), strain variance is reduced when the variance in the time delay estimate is minimized. The elastographic resolution (depends on both \( T \) and \( \Delta t \)) is reduced with an increase in \( T \). Large overlapping windows also generate correlated errors that bias the strain estimate. In addition, an optimal window size exists, where the strain estimation variance is minimum, with an increase in the variance observed as the window size is increased or decreased [4, pp. 122,29]. Therefore the strain estimation variance cannot simply be reduced in the limit as \( T \rightarrow \infty \), since \( \sigma^2_s \) also increases.
APPENDIX B

Development of the Strain Filter

Development of the strain filter concept is based on obtaining the tightest bound on the variance of the strain estimator that includes all noise sources. Since the strain estimator uses time-delay (displacement) estimation to compute strain (see Appendix A, Eq. (A-1)), the vast literature on TDE can be adapted for the strain estimation problem.

Noise Sources

The CRLB is the most commonly used lower bound on time-delay variance [19–22]. While sonographic and quantization noise contributions are accounted for in the expression for the CRLB through the SNR term (sonographic SNR denoted by SNRₜ) [19–22], the added effect of decorrelation on the variance of the time delay estimate has been modeled recently by Walker and Trahey [23], using the correlation coefficient. The authors also indicate that TDE performance can be worse than the CRLB at poor SNRs (<15 dB) and low correlation coefficient values (<0.5).

Signal decorrelation, a significant source of error in the displacement estimate, increases rapidly with tissue compression. Decorrelation errors are caused by the relative displacement of the scatterers in all three dimensions due to tissue compression. Decorrelation errors increase with tissue strain [3,5,8], causing a decay in the values of the correlation coefficient [3,5]. Expressions for the rf correlation coefficient before and after temporal stretching, have been derived by Varghese and Ophir in [5], for a cosine-pulse modulated by a Gaussian envelope. The expressions for the correlation coefficient before and after temporal stretching are presented in Appendix C.

Expressions for the CRLB [19–23] have been derived for flat bandlimited signal and noise spectra. However, the correlation coefficient in [5,24] is derived for a Gaussian-shaped spectrum rather than a rectangular spectrum. A reasonable approximation was obtained by Céspedes et al. [25] using a rectangular spectrum centered at the Gaussian center frequency with the same mean square amplitude value as the Gaussian spectrum. The equivalent noise spectral bandwidth [26, pp. 141] is defined by:

\[
B = \frac{\int_{0}^{\infty} P(f) \, df}{P(f)_{\text{max}}} = \frac{1}{\sqrt{2\pi\sigma}}, \tag{B-1}
\]

where \(B\) is the bandwidth of a rectangular spectrum with the same total power and peak amplitude as the Gaussian pulse spectrum \(P(f)\).

Signal decorrelation may be modeled as a noise process that reduces the composite signal-to-noise ratio (SNRₜ) in the echo-signal. SNRₜ is a combination of the constant sonographic noise level (SNRₜ) and a varying component due to signal decorrelation, and is given by [19,20]:

\[ \text{SNR}_t = \text{SNR}_S + \text{SNR}_D \]
\[ SNR_C = \frac{SNR_S SNR_p}{1 + SNR_S + SNR_p}, \]  

where \( SNR_p = \rho / (1 - \rho) \) converts the correlation coefficient to an SNR measure. This relationship, independently derived by Friemel [27] and Céspedes et al. [28], enables the use of \( SNR_C \) in CRLB expressions in the literature that involve signal decorrelation. This allows incorporation of signal processing algorithms into the strain filter (discussed below), as long as their effect on the signal correlation can be computed. From Eq. (B-2), we observe that \( SNR_C \) will always be bounded by the smallest value of either \( SNR_S \) or \( SNR_p \).

Increasing signal decorrelation errors with tissue strain reduce \( SNR_C \), causing the strain estimation variance to exceed the CRLB (as also noted by Walker and Trahey [23]), thereby necessitating the need for a more advanced lower bound to predict the variance of the strain estimator. The modified Ziv-Zakai lower bound proposed by Weinstein and Weiss [19,20] is used to obtain an accurate lower bound.

The Ziv-Zakai Lower Bound (ZZLB) on Time-Delay Estimation

Weinstein and Weiss present plots of the lower bound of the time delay variance versus the postintegration SNR [19,20] (defined as a product of the bandwidth, data window length and the \( SNR_C \)), that divide the postintegration SNR domain into three distinct regions (low, moderate and high). Since \( SNR_C \) is the only parameter in the expression for the postintegration SNR that varies with tissue strain, its value determines the appropriate lower bound.

The CRLB is applicable only to high postintegration SNR situations (at low strains). In this region, the time-delay estimation is subject only to local errors (ambiguity-free mode of operation). At moderate SNR values, the lower bound exceeds the CRLB, and obeys the Barankin bound. In this region, ambiguities in the signal phase cannot be resolved; however, an estimate of the time-delay estimate may still be obtained using the correlation between signal envelopes. At low postintegration SNR values, the lower bound approaches a constant level. In this region, both envelope and phase ambiguities exist, and the time delay cannot be estimated correctly. The thresholds separating these three regions are determined by the value of the postintegration SNR [19,20]. The above combination of lower bounds on the variance of the time-delay estimator is referred to as the Ziv-Zakai lower bound [19,20]. The ZZLB provides the tightest bound on the variance of the time-delay estimator.

The Strain Filter in Elastography

A measure of elastographic image quality was described [3,4] in terms of the mean-to-standard deviation ratio (\( SNR_e \)) of the elastogram:

\[ SNR_e = \frac{m_s}{\sigma_s}, \]
where \( m_s \) and \( \sigma_s \) denote the mean and standard deviation of the strain estimates in a region of uniform elasticity. The upper bound of the SNR is obtained when the total tissue strain \( (s_t) \) and the lower bound on the strain estimation standard deviation \( (\sigma_{ZLZB}) \) are substituted in Eq. (B-3):

\[
\text{SNR}_{e}^{\text{UB}} = \frac{s_t}{\sigma_{ZLZB}}.
\]  

The modified ZLZB expression for the strain estimation variance [10] is given by:

\[
\sigma_{ZLZB}^2 \geq \begin{cases} 
\frac{(sT)^2}{6T\Delta t} & BT \text{ SNR}_C \ < \gamma \\
2\sigma_{BB}^2 & \gamma \ < \ BT \text{ SNR}_C \ < \delta \\
2\sigma_{CRLB}^2 & \delta \ < \ BT \text{ SNR}_C \ < \eta \\
\frac{2.76}{T\Delta t} & \eta \ < \ BT \text{ SNR}_C 
\end{cases}
\]  

where \( T \) is the length of the temporal window, \( \Delta t = t_{1a} - t_{1b} \) is the overlap between successive windows, \( \sigma_{CRLB}^2 \) represents the CRLB, and \( \sigma_{BB}^2 \) represents the Barankin bound. The quantity \( BT \text{ SNR}_C \) is referred to as the postintegration SNR. Eq. (B-5) shows the three distinct operating regions for \( \sigma_{ZLZB}^2 \), depending on the value of \( BT \text{ SNR}_C \). The strain variance is computed from the TDE variance using the expression derived by Walker and Trahey [23], as long as the ZLZB coincides with the CRLB. The Barankin bound exceeds the CRLB by a factor of \( 12(f_0/B)^2 \) [19,20]. A distinct threshold region is observed between the CRLB and the Barankin bound, however the variance increases exponentially in this threshold region. Accurate estimation of the strain is possible only within the CRLB. The threshold points used in Eq. (B-5) are defined as follows [10]:

\[
\eta = \frac{2}{T\Delta t} \frac{6}{\pi^2} \frac{(f_0)^2}{B} \left[ \varphi^{-1} \left( \frac{B^2}{24(f_0)^2} \right) \right]^2
\]

\[
\zeta = \frac{2}{T\Delta t} \frac{2.76}{\pi^2} \frac{(f_0)^2}{B}
\]

\[
\delta = \frac{1}{T\Delta t} \zeta
\]

\[
\gamma = \frac{2}{T\Delta t} \ 0.46,
\]

where \( f_0 \) is the center frequency, \( B \) is the rectangular bandwidth, \( \varphi^{-1}(y) \) is the inverse of

\[
\varphi(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\mu^2/2} \ d\mu,
\]
and \((\zeta/2)\varphi(\sqrt{\zeta/2}) = (6/BsT)^2\), which has two solutions. The larger value of \(\zeta\) is used to compute the threshold. When \(\eta < BT\) SNR\(_C\), the ZZLB coincides with the CRLB, which is the ambiguity-free region. If \(\delta < BT\) SNR\(_C < \delta\), the ZZLB coincides with the Barankin bound, where phase ambiguities increase the TDE variance. Finally, when \(BT\) SNR\(_C < \gamma\), the lower bound is characterized by the constant variance level of \((sT)^2/6T\Delta t\), which corresponds to the variance of a random variable uniformly distributed between \([-sT/2, sT/2]\).

The variation of SNR\(_C^{UB}\) (Eq. (B-4)) with tissue axial strain is defined as the strain filter. Three distinct regions constitute the strain filter, which depends on the appropriate lower bound that contributes to \(\sigma^2_{ZZLB}\) (Eq. (B-5)). Decorrelation errors increase with tissue strain [4], causing \(\sigma^2_{ZZLB}\) to move from the CRLB to the Barankin bound or the constant variance level, as shown in Eq. (B-5). The three distinct regions in Eq. (B-5) are observed in plots of the strain filter, as shown by the curves in figure 1. When the lower bound coincides with the Barankin bound, the performance of the strain filter drops sharply, with a further drop in performance observed when the variance coincides with the constant variance level.

The range of strains that can be reliably depicted in an elastogram determines the dynamic range of the strain filter, which is defined as

\[
DR = 20 \log_{10} \left[ \frac{s_{max}}{s_{min}} \right],
\]  

(B-7)

where \(s_{max}\) is the maximum strain and \(s_{min}\) is the minimum strain at a specified SNR\(_e\) level in the strain filter. The quantity \(s_{min}\) also defines the sensitivity of the strain filter.

APPENDIX C

Derivation of the Correlation Coefficient

Elastography uses a pair of rf echo signals (pre- and postcompression) to obtain the strain estimate. The pre- and the postcompression echo signals are obtained from a tissue model that consists of a collection of randomly distributed weak scattering centers that interact with the ultrasound pulse only once (multiple scattering is neglected) as it propagates through the tissue. The rf echo-signal before compression along the axial direction is given by

\[
r_1(x) = e(x) * p(x) + n_1(x),
\]  

(C-1)

where \(e(x)\) is the scattering function of the elastic tissue scattering particles, \(p(x)\) is the impulse response of the system, \(n_1(x)\) denotes the uncorrelated random noise, and \(*\) denotes the convolution operation.

The randomly distributed elastic tissue scattering particles displace in all the three dimensions (axial, lateral and elevational) under tissue compression. Under the assumption of a semi-infinite tissue medium, uniform strain distribution and under slip boundary conditions, tissue displacement along the lateral and elevational directions are small when
compared to the axial displacement. The rf echo signal after compression along the axial direction is given by

\[ r_2(x) = e \left( \frac{x}{\alpha} \right) \ast p(x) + n_2(x), \]  

(C-2)

where \( r_1(x) \) and \( r_2(x) \) denote the pre- and postcompression rf echo signals respectively, and \( \alpha \) depends on the applied tissue compression and is given by \( \alpha = 1 - s \), where \( s \) is the applied strain. The correlation coefficient indicates the similarity between the pre and the postcompression of rf echo-signals and is defined as

\[ \rho_{12} = \frac{C_{12}}{\sqrt{R_{11}R_{22}}}, \]  

(C-3)

where \( C_{12} \) is the maximum value of the crosscorrelation function, \( R_{11} \) and \( R_{22} \) are the corresponding maximum values of the autocorrelation functions.

Temporal stretching of the postcompression echo signal, significantly improves the correlation between the pre- and the postcompression signals [4,5,7]. The postcompression signal is linearly restretched by the factor by which the scatterers were compressed. This operation on the postcompression echo signal can be expressed as:

\[ r_3(x) = e(x) \ast p(\alpha x) + n_3(x), \]  

(C-4)

where \( r_3(x) \) denotes the temporally stretched postcompression rf signal. The correlation coefficient between the pre- and the stretched postcompression signal is given by:

\[ \rho_{13} = \frac{C_{13}}{\sqrt{R_{11}R_{33}}}, \]  

(C-5)

where \( C_{13} \) is the maximum value of the crosscorrelation function with temporal stretching, and \( R_{33} \) is the maximum value of the autocorrelation function of the stretched postcompression signal. The maximum value of the correlation coefficient before and after temporal stretching has been derived by Varghese and Ophir [5]. The rf correlation coefficient before temporal stretching can be expressed as

\[ \rho_{12} = \frac{\alpha \sqrt{2\alpha}}{(1 + e^{-((\sigma_{k0})^2/2)((1-\alpha)^2/(\alpha^2+1)))}} \left( e^{-((\sigma_{k0})^2/2)((1-\alpha)^2/(\alpha^2+1)))} + e^{-((\sigma_{k0})^2/2)((1+\alpha)^2/(\alpha^2+1)))} \right). \]  

(C-6)

where \( \rho_{12} \) is the peak correlation coefficient before temporal stretching, \( k_0 = 2\pi/\lambda_0 \) is the wave number, where \( \lambda_0 \) is the wavelength of the ultrasound pulse, and \( \sigma \) is the standard deviation of the Gaussian pulse. The rf correlation coefficient after temporal stretching can be expressed as:
where \( \rho_{13} \) is the peak correlation coefficient after temporal stretching. When \( \alpha = 1 \) (no tissue compression), the correlation coefficients \( \rho_{12} = \rho_{13} = 1 \), yielding the best possible match between the pre- and postcompression echo rf signals. Temporal stretching of the post-compression rf signal reduces signal decorrelation and improves the performance of the strain estimator. Signal decorrelation also reduces with an increase in the pulse bandwidth, or a decrease in the axial duration of the pulse [24]. A one-dimensional tissue model that considers signal decorrelation only due to the axial displacement of the tissue scatterers is used to obtain the above expressions for the correlation coefficient. Eqns. (C-6) and (C-7) are valid for all values of tissue strain.

APPENDIX D

Multicompression Elastography

Multicompression elastography performs strain imaging over a number, \( N \), of small applied strain increments, \( s_s \), that minimizes signal decorrelation and allows accurate temporal stretching of the post-compression signal. These applied strain increments add up to a total tissue strain \( s_t = Ns_s \). The strain estimates obtained in multicompression elastography can be combined either by averaging, or by accumulation. Both methods produce the same improvement in the elastogram.

The averaging procedure reduces the standard deviation (the notation for the standard deviation uses \( \sigma_{ZZLB} \) which is the Ziv-Zakai lower bound; refer to Appendix B for details) of the averaged elastogram by \( \sqrt{N} \) (assuming that the strain estimates are uncorrelated). Thus there is a net increase (using the applied strain increment) of the SNR\(_e\) by a factor of \( \sqrt{N} \), viz.

\[
\text{SNR}^\text{UB} = \frac{s_s}{\sigma_{ZZLB}/\sqrt{N}} = \frac{s_s\sqrt{N}}{\sigma_{ZZLB}}. \tag{D-1}
\]

The accumulation method add all strain estimates to obtain the total strain. The standard deviation of the accumulated strain increases by a factor of \( \sqrt{N} \) (assuming that the strain estimates are uncorrelated). However, the strain in each step is \( s/N \), thus resulting in a net increase of \( \sqrt{N} \), of the SNR\(_e^\text{UB}\) for the multiple step case, viz.,

\[
\text{SNR}^\text{UB} = \frac{s_t}{\sqrt{N}\sigma_{ZZLB}} = \frac{s_s\sqrt{N}}{\sigma_{ZZLB}}. \tag{D-2}
\]

Hence, both methods of multicompression elastography increase the SNR\(_e^\text{UB}\) by a factor of \( \sqrt{N} \) for the applied strain increment. Assuming that signal decorrelation does not occur for a single total applied strain increment of \( s_t \) (normalized correlation coefficient = 1),
multicompression would reduce the maximum achievable \text{SNR}_c^\text{UB} by a factor of $\sqrt{N}$. However, since we are always faced with the deleterious effects of signal decorrelation (normalized correlation coefficient $< 1$) on the performance of the strain estimator, multicompression elastography is required to image the entire range of strains in tissue.

REFERENCES


