THE NONSTATIONARY STRAIN FILTER IN ELASTOGRAPHY: PART II. LATERAL AND ELEVATIONAL DECORRELATION

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Abstract—The nonstationary evolution of the strain filter due to lateral and elevational motion of the tissue scatterers across the ultrasound beam is analyzed for the 1-D cross-correlation-based strain estimator. The effective correlation coefficient that includes the contributions due to lateral and elevational signal decorrelation is used to derate the upper bound of the signal-to-noise ratio in the elastogram (SNR) predicted by the ideal strain filter. In the case of an elastically homogeneous target, if the transducer is on the axis of symmetry in the elevational direction, the motion of the scatterers out the imaging plane is minimized. In addition, the ultrasound beam along the elevational direction is broader, allowing scatterers to stay longer within the beam during tissue compression. Under these conditions, lateral signal decorrelation becomes the primary contributor to the nonstationary behavior of the strain filter. Both the elastographic SNR and the dynamic range are reduced, with an increase in lateral decorrelation. Finite element simulations and phantom experiments are presented in this paper to corroborate the theoretical strain filter. The nonstationary behavior of the strain filter is reduced by confining the tissue in the lateral direction (minimizing motion of tissue scatterers), thereby improving the quality of the elastogram. © 1997 World Federation for Ultrasound in Medicine & Biology.

Key Words: Elasticity, Imaging, Ultrasound, Elastography, Strain, Elastogram, Noise, Lateral motion, Elevational motion, Nonstationarity.

INTRODUCTION

Several tissue-stiffness measurement techniques have been reported in the past few years (Ophir et al. 1991; Parker et al. 1990, 1997; O'Donnell et al. 1994; Emelianov et al. 1995; Sarvazyan 1993; Muthupillai et al. 1995; Plewes et al. 1994; Lewa et al. 1995; Fowlkes et al. 1995). In this paper, we are interested in elastography proposed by Ophir et al. (1991). This technique has been successfully applied in vivo for the evaluation of breast lesions, and showed potential for improving the ability of radiologists to distinguish benign from malignant masses (Carra et al. 1997).

Recently, elastography was described as a cascade of two distinct processes (Ophir et al. 1997). The first process involves the mapping of the distribution of local elastic moduli in the target into a distribution of local longitudinal strains. This process is governed by the theory of elasticity as applied to a particular experimental setup under specific boundary conditions, and some assumptions. Because this process involves errors due to the simplified mechanical model used, artifacts such as target-hardening, shadowing and limited contrast-transfer efficiency are encountered (Ophir et al. 1996; Ponsekanti et al. 1995; Kallel et al. 1996). Solution of the inverse problem in elastography (obtaining tissue modulus images from the strain images or elastograms) reduces these artifacts (Kallel and Bertrand 1996; Skovoroda et al. 1995; Sumi et al. 1995).

The second process involves the generation of the strain image (elastogram) from ultrasonically estimated values of local strains. Limitations of the ultrasound system (such as time-bandwidth product, center frequency, beamwidth and sonographic SNR), as well as the algorithms used to process the signals, cause additional corruption of the tissue strain information. The strain filter (SF) concept (Varghese and Ophir 1997b) describes the elastographic image parameters, such as the attainable elastographic SNR, strain dynamic range (width of the SF at a given SNR), and sensitivity (smallest measurable strain) at a given value of the resolution.
(window length). Note that, in practice, the data used by the inverse problem to reconstruct the tissue stiffness distribution are those produced by this process. Therefore, a complete understanding of these two interdependent processes is the key for improvement of image quality in elastography.

In elastography, tissue axial displacements induced by an external quasi-static compression are measured using a 1-D cross-correlation technique applied to RF echo signals. The performance of the displacement estimator has been analyzed in many fields (Carter, 1993). However, most of these models have been developed for pure displacement (time-delay estimation) but, in elastography, we deal with additional signal distortions due to the physical deformations of the tissue that results from the applied compression. Recently, these models have been adapted to the strain estimator in elastography by including the effects of signal distortions (Varghese and Ophir 1997b; Cespedes et al. 1995; Cespedes 1993; Bilgen and Insana 1996, 1997a, 1997b; Alam and Ophir 1997).

In elastography, the postcompression RF echo signal is not exactly a delayed version of its precompression signal. Indeed, due to the applied strain, the tissue scatterers move relative to each other, contributing to differences in the resulting pre- and postcompression acoustic speckle patterns. This difference is referred to as speckle decorrelation (Meunier and Bertrand 1995; Cespedes et al. 1995). Therefore, to use the models developed for pure time-delay estimation, the postcompression signal is modeled as a pure delayed version of its corresponding precompression signal, plus an additional noise term to model signal decorrelation. Signal decorrelation was modeled using the value of the correlation coefficient in the strain filter formulation (Varghese and Ophir 1997b).

The approach proposed by Bilgen and Insana (1997a, 1997b) to measure the elastographic imaging parameters is quite similar to the approach proposed by Varghese and Ophir (1997b). Bilgen and Insana (1997a, 1997b) derived closed-form expressions for the variance of the estimated displacement and strain based on a Taylor’s series expansion. These expressions are a function of the imaging system and the signal-processing parameters, and are valid only in cases with nonambiguous detection of the cross-correlation peak. The estimated strain variance is then used to predict a noise-figure measure defined as the ratio of the elastogram to the sonographic SNR. When plotted as a function of applied strain, the elastographic noise-figure is similar to a strain titter.

In elastography, tissue scatterers undergo a complex motion pattern that depends on the boundary conditions and elastic properties of the tissue. However, to make the problem tractable, it is assumed that the tissue deforms uniformly under an external compression. The tissue is modeled mechanically as a spring (perfectly compressible), and acoustically as a 1-D random distribution of scatterers. In practice, soft tissues are incompressible (Parker et al. 1990; Sarvazyan et al. 1993) and, thus, the motion of their scatterers occurs in a 3-D space. Therefore, the previously used simplifying assumption may result in an overestimation of the elastogram SNR, as predicted by Varghese and Ophir (1997b) using the strain filter and by Bilgen and Insana (1997b).

In this paper, we formulate the nonstationarity in the strain filter, in the lateral and elevational directions, by incorporating the effects of lateral and elevational signal decorrelation. This formulation is based on the use of a full 3-D model for both the tissue scatterers and the ultrasound beam to completely model the motion of the tissue scatterers under a quasi-static compression. In such formulation, the effective correlation coefficient has been derived as the product of the axial, lateral and elevational correlation coefficients by Kallel and Ophir (1997). In the previous formulations of the strain filter (Varghese and Ophir 1997b; Varghese et al. 1996), the acoustic model used to derive the correlation coefficient was a 1-D model that did not include the effects of lateral and elevational decorrelation.

For an elastically homogeneous, isotropic target, when the array transducer is on the axis of symmetry of the target in the elevational direction, the motion of the scatterers in that direction is close to zero, thereby reducing the elevational correlation coefficient to a value close to 1. The effect of lateral decorrelation, therefore, contributes predominantly to the nonstationary behavior of the strain filter (Kallel and Ophir, 1997).

This paper is organized as follows. In the next section, we describe the theory for the computation of the effective correlation coefficient using a 3-D acoustic model. The nonstationarity of the strain filter in the lateral direction and the subsequent improvement of SNR with lateral confinement of tissue are also discussed. Then, simulation results using finite-element analysis are presented. Experimental results that corroborate the theoretical formulation and simulation results follow. Finally, the contributions of this paper are summarized.

**THEORY**

The axial correlation coefficient used to model signal decorrelation (Varghese and Ophir 1997a; Varghese et al. 1996c) varies with applied strain and is itself also dependent on the window length (Z) used to compute the normalized cross-correlation function. In addition, further derating of the correlation coefficient occurs due to lateral and elevational motion of tissue scatterers across the ultrasound beam. In this paper, we obtain an expression for the effective correlation coefficient that accounts
for axial, lateral and elevational signal decorrelation using a 3-D acoustic model.

Expression for the effective correlation coefficient

For an elastically homogeneous medium subjected to a uniform applied strain with perfect slip boundary conditions, we assume a separable 3-D PSF at the focus of the transducer, and a constant motion of the tissue scatterers within the beam. For a fully developed tissue speckle, the effective correlation coefficient between the pre- and postcompression RF A-lines which includes all the contributions due to axial, lateral and elevational signal decorrelation, has been derived by Kallel and Ophir (1997), and is given by:

\[
\hat{\rho} = \frac{C_L(u(x_0))}{C_L(0)} \times \frac{C_L(v(y_0))}{C_L(0)} \times \hat{\rho}_x \tag{1}
\]

where \(C_L(u(x_0))\) and \(C_L(v(y_0))\) are the autocovariance functions of the elevational and lateral impulse-response functions, respectively, and the ratios

\[
\frac{C_L(u(x_0))}{C_L(0)} \quad \text{and} \quad \frac{C_L(v(y_0))}{C_L(0)}
\]

represent the elevational and lateral correlation coefficients, respectively; \(u(x_0)\) and \(v(y_0)\) are the lateral and elevational tissue scatterer motions (constants) across the ultrasound beam; \((x_0,y_0)\) define the elevational and lateral position of the ultrasound beam relative to the scanned target. This expression describes the effective correlation coefficient as a product of the respective axial (\(\hat{\rho}_x\)), lateral (\(\hat{\rho}_l\)) and elevational (\(\hat{\rho}_e\)) correlation coefficients. The effective correlation coefficient is therefore given by:

\[
\hat{\rho} = \hat{\rho}_l \times \hat{\rho}_e \times \hat{\rho}_x \tag{2}
\]

In the next sections, the axial, lateral and elevational correlation coefficients are derived for a Gaussian-shaped pulse and ultrasonic beam profile.

Expression for the effective axial correlation coefficient

The effective axial correlation coefficient depends on tissue strain, the finite size of the window used for the cross-correlation analysis and the sonographic noise. A closed-form expression for the effective axial correlation coefficient has been derived recently by Varghese et al. (1998). The expression for the effective correlation coefficient is modeled as a product of the maximum value of the correlation coefficient and a derating factor. The derating factor accounts for the decay in the correlation coefficient with increased window length. The effective axial correlation coefficient is expressed as follows:

\[
\hat{\rho}_x = \rho_{PEAK} \times M. \tag{3}
\]

The term, \(\rho_{PEAK}\), as discussed in the literature, calculates the peak correlation coefficient due to the 2-point ensemble averaging. The peak value of the correlation coefficient is derived in (Bilgen and Insana, 1996), and is given by:

\[
\rho_0 = \frac{1}{2} \left( \frac{L_z^2 + a^2 \sigma_z^2}{L_z^2 + L_n^2} \right) \exp \left( \frac{\xi^2}{\eta^2} \right) \tag{4a}
\]

where \(L_z\) represents the correlation length of the ultrasound pulse, \(k_o\) is the wave number of the pulse, \(L_m\) represents the correlation length of the tissue reflectivity profile centered at \(k_m\), and where:

\[
a = \frac{1}{1 - s} \tag{4b}
\]

\[
\eta^2 = L_{z_f}^2 + \frac{1 + a^2}{2} \frac{L_n^2}{L_{z_f}^2} \tag{4c}
\]

\[
\xi = \omega_t L_z^2 + \frac{1 + a}{2} \omega_t L_{z_f}^2 \tag{4d}
\]

The factor \(M\) in eqn (3) is due to the finiteness of the window length that derates the value of the maximum correlation coefficient. This derating function is derived in (Varghese et al. 1998), and is given by:

\[
M = \frac{1}{Z} \int_{-Z}^{Z} \exp \left( -\frac{((1 - a)z)^2}{4\eta^2} \right) \cos \left( \frac{\xi((1 - a)z)}{\eta} \right) dz. \tag{5}
\]

Evaluating the integral yields the derating function as:

\[
M = \sqrt{\frac{\pi \eta}{1 - a}} \exp \left( -\frac{\xi^2}{\eta^2} \right) \left( \text{erf} \left[ \frac{((1 - a)Z + i \xi \eta)}{4 \eta} \right] - \text{erf} \left[ \frac{((1 - a)Z + i \xi \eta)}{4 \eta} \right] \right) \tag{6}
\]

\[1\] This assumption is valid for small tissue strain and when the beam is narrow. In elastography, such assumption is reasonable.
Fig. 1. Variation of the axial RF correlation coefficient for different window lengths plotted as a function of the tissue strain. Observe that the correlation coefficient decays rapidly with an increase in window length. The axial correlation coefficients were obtained using the following parameters: $L_x = L_m = 0.08 \text{ mm}$, center frequency of 7.5 MHz.

where the error function is defined as follows:

$$\text{Erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)dt.$$  \hspace{1cm} (7)

For zero strain $s = 0$, $a = 1$ and one can show that $M = 1$ in the limit. Nonzero strains, on the other hand, yield $M < 1$ and consequently reduce the correlation coefficient $p$. Around the axis of the symmetry of the homogeneous target where the scatterers lateral and elevational motions are close to zero, both the pre- and the postcompression signals are completely correlated. The parameter $a$ is the strain factor defined in terms of either the actual tissue strain $s$, $a = \frac{1}{1-s}$ for $s << 1$ or the residual strain $\delta s$, $a = 1 + \delta s$ if the postcompression echo signal is stretched back temporally by the average strain. In either case, $a$ is not constant and changes continuously with position in the tissue. However, in the derivation of the axial correlation coefficient, we treat the strain as locally constant within each window because any continuous profile can be approximated by a piece-wise variation. In this paper, the temporal stretching operation is used to obtain the nonstationary strain filter. The variations in the effective axial correlation coefficient with tissue strain for different window lengths are plotted in Fig. 1. Note that the larger window lengths can be used to estimate strain only for very small strain because the effective axial correlation coefficient drops rapidly with increased tissue strain.

The model predicting the axial correlation coefficient [eqn (3)] is derived assuming noise-free RF signals. In practice, depending on the noise level, a limit on the smallest window size exists.

Expression for lateral and elevational correlation coefficients

We assume a Gaussian beam profile is given by:

$$h_s(y) = \frac{1}{\sqrt{2\pi L_y}} \exp(-y^2/2L_y^2).$$  \hspace{1cm} (8)

where $L_y$ is a beamwidth parameter derived from the full-width half-maximum (FWHM) of the Gaussian function given in eqn (8) (beamwidth $= \text{FWHM} \approx 2.35 L_y$). The autocovariance of the above impulse response has been derived by Wagner et al. (1983), and is given by:

$$C_{tl}(\Delta y) = K_s \exp(-\Delta y^2/4L_y^2).$$  \hspace{1cm} (9)

Therefore, the correlation coefficient in the lateral direction, using eqn (9), can be written as:

$$\rho_L = \frac{C_{l}(v(y_0))}{C_{l}(0)} = \exp\left(-\left(\frac{v(y_0)}{4L_y^2}\right)^2\right).$$  \hspace{1cm} (10)

Assuming perfect slip boundary conditions, the lateral displacement field for a uniform applied strain $(s)$ is given by (Kallel and Ophir, 1997):

$$v(y_0) = -svy_0$$  \hspace{1cm} (11)

where $\epsilon$ is the Poisson’s ratio of the tissue medium. Because soft tissues are incompressible, we use a Poisson’s ratio $\epsilon = 0.5$. The lateral correlation coefficient can therefore be expressed as:

$$\rho_L = \exp\left(\frac{(-svy_0)^2}{4L_y^2}\right).$$  \hspace{1cm} (12)

Similarly, using the same assumptions for the beam in the elevational direction, the elevational correlation coefficient can be expressed as:

$$\rho_E = \exp\left(\frac{(-svy_0)^2}{4L_z^2}\right).$$  \hspace{1cm} (13)

When the transducer is on the axis of symmetry of the homogeneous target in the elevational direction, the motion of the scatterers in that direction is close to 0, thereby reducing the elevational correlation coefficient to a value close to 1. Therefore, the lateral decorrelation is the dominant contributor to the nonstationary behavior of
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Fig. 2. Variation of the lateral correlation coefficient at different lateral positions of the ultrasound beam, plotted as a function of tissue strain. Observe that the correlation coefficient decays rapidly with an increase in lateral position (also equivalent to an increase in lateral displacement). The lateral correlation coefficients were obtained using the following parameters: $L_c = 0.29$ mm (beamwidth $= 2.35 L_c = 0.68$ mm) and $v = 0.5$.

The nonstationarity of the strain filter with lateral position

The variation in the strain filter incorporating the effects of lateral decorrelation is shown in Fig. 4, where a family of strain filters is plotted at different lateral positions of the ultrasound beam relative to the axis of symmetry of the target (tissue) generated for a 1-mm axial window length. The highest strain filter is obtained for 0 lateral displacement (ultrasound beam around the axis of symmetry of the target), with the reduction in the SNR$_e$ and dynamic range observed with an increase in the lateral displacement (beam approaching the edges of the linear array transducer). This reduction in the strain filter is due to the combined effects of the axial and lateral signal decorrelation (causing a reduction in the value of the effective correlation coefficient) as illustrated in Fig. 3. Figure 5 shows a 3-D plot of the nonstationary variation in the strain filter with lateral ultrasound beam position, tissue strain and elastographic SNR$_e$ plotted along the 3 axes. The strain filter is derated,

![Graph showing variation in strain filter with lateral position](image)

![Graph showing variation in effective correlation coefficient with lateral position](image)
Fig. 5. A 3-D plot of the strain filter illustrating the nonstationary variation with lateral position. Tissue strain in dB is plotted along the x-axis, lateral position along the y-axis and SNR along the z-axis. The maximum SNR of the strain filter reduces with lateral position. The strain filters were obtained for the following parameters: \(k = 30.6 \text{ rad/mm} \) (7.5-MHz center frequency and 1540 m/s for the speed of sound), \(L = 0.08 \text{ mm} \) (50% bandwidth), \(Z = 1 \text{ mm} \), and \(\Delta z = 0.5 \text{ mm} \).

with a sharp decline in the SNR and dynamic range observed at large lateral displacements (near the linear array transducer edges). Plotting the strain filter in this manner allows the visualization of the entire range of tissue strains observable at different lateral positions in the elastogram. Note that the maximum dynamic range and elastographic SNR are obtained along the axis of the transducer (center of the elastogram), with a progressive reduction in the elastographic SNR along the lateral axis.

Figure 6 illustrates the nonstationary variation in the strain filter using a 3-mm window length. Note that, for larger window lengths, the effect of axial signal decorrelation is more pronounced when compared to lateral signal decorrelation (note the variation in the correlation coefficient curves in Figs. 1 and 2). Increasing the axial window length increases the information content in the signal; however, signal decorrelation effects (primarily axial signal decorrelation), especially at large tissue strains, contribute to the derating of the strain filter. The contributions of lateral and elevational signal decorrelation are more pronounced with an increase in the resolution (corresponding to a decrease of the beam width).

The variation in the SNR vs. lateral ultrasound beam position relative to the axis of symmetry of the target (0 mm) is plotted in Fig. 7 for various values of tissue strain. Observe that the SNR is maximum when the ultrasound beam is near the axis of symmetry of the target (0 lateral displacement), and decreases with an increase in the lateral ultrasound beam position. The maximum value of the SNR is obtained at a strain value of 0.5%, with a reduction in the value obtained for both

Fig. 6. Nonstationary variation in the strain filter for different lateral positions. Lateral decorrelation increases with an increase in the beam lateral position (increased lateral tissue scatterers motion) (see Fig. 1), reducing the strain estimation performance with lateral position. The strain filters were obtained for the following parameters: \(k = 30.6 \text{ rad/mm} \) (7.5-MHz center frequency and 1540 m/s for the speed of sound), \(L = 0.08 \text{ mm} \) (50% bandwidth), \(Z = 3 \text{ mm} \), and \(\Delta z = 0.5 \text{ mm} \).
The quality of the elastogram is reduced along the lateral direction by the nonstationarity of the strain filter, as illustrated in Figs. 4–7. To maintain a constant \( \text{SNR}_e \) in the entire elastogram, techniques to minimize the lateral motion of the tissue scatterers and, hence, lateral decorrelation are of paramount importance. Confining the tissue along the lateral direction is one such technique that reduces lateral motion of the tissue scatterers.

In this case, the elevational tissue motion (out-of-plane motion) is doubled. Fortunately, in this direction, the beam is wide and generally the transducer is on the target axis of symmetry where, as discussed before, the scatterers remain within the acoustical scanning plane. Theoretically, this would correspond to the flattening of the curves shown in Fig. 7, about their respective maximum \( \text{SNR}_e \) values. Lateral confinement of the tissue reduces the effective signal decorrelation to include just the effects of axial signal decorrelation. The effective correlation coefficient in the strain filter formulation is, therefore, constant for all lateral positions in the tissue (along the scanning plane). The effect of lateral confinement contributes to making the strain filter stationary in the lateral direction. The reduction of the lateral nonstationarity in the elastogram \( \text{SNR} \) may also be obtained by tracking the speckle motion along the lateral direction as suggested by Insana et al. (1996), using an algorithm called sum of absolute differences (SAD) (Bohs, 1991).

Another interesting case is the prediction of the strain estimation results, obtained by confining tissue along the elevational direction and allowing tissue motion only along the axial and lateral direction. In this case, the motion in the lateral direction is doubled, as observed from Fig. 8. Comparing Figs. 7 and 8, we observe the increased effect of lateral signal decorrelation on the variation of \( \text{SNR} \), with lateral displacement. As shown by Fig. 8, for a tissue strain of 3%, the
transition of the strain estimation variance from the CRLB to the Barankin bound occurs when the beam is at ± 8 mm relative to the target axis of symmetry (0 mm). Notice that, in such a case, if the speckle motion is accurately tracked in the lateral direction using the SAD algorithm, both the lateral and elevational contribution to the nonstationarity of the SNR will be reduced.

In the next section, the model predicting the effective axial correlation coefficient is first validated experimentally. Second, both simulation and experimental results are used to compare the performance of the strain estimator for a 1% tissue strain. Strain estimations under 3 different boundary conditions are utilized to validate the theoretical results presented in this section. The theoretical predictions are obtained under the boundary conditions of: 1. No tissue confinement; 2. tissue confinement in the lateral direction; and 3. tissue confinement in the elevational direction, and the results are presented in Fig. 9. Simulation results and experimental analysis using a uniformly elastic phantom are also provided in the subsequent sections, to corroborate the theoretical results presented in this section.

RESULTS

Effective correlation coefficient: experimental validation

In this section, we show the lateral decorrelation effects using an elastically homogeneous gel phantom made of a mixture of gelatin, water, and small acoustic scatterers (graphite flakes) (Hall et al. 1996). To have a uniform axial strain distribution, the phantom was compressed using a large rectangular plate, in which the linear array transducer was inserted. Furthermore, we attempted to have perfect slip boundary conditions by lubricating both the upper and lower surfaces of the phantom with corn oil. The ultrasound system used for this experiment is a Diasonics Spectra II real-time linear array scanner (Diasonics Ultrasound, Santa Clara, CA) that operates with dynamic receive focusing and a single transmit focal zone centered at 30 mm. A center frequency of 7.5 MHz is used. The digitized data were collected from a 40 × 40 mm ROI (starting at a depth of 5 mm under the transducer), centered around the transmit focus and for an applied strain ranging from 0.4 to 4% with steps of 0.2%. In each trial, the ROI consisted of 100 A-lines. The overall elastography system was fully described in previous work (Céspedes 1993).

Figure 10 shows the correlation coefficient at the peak of the normalized cross-correlation function computed between 2.5-mm precompression and stretched postcompression echo segments. These segments are taken in the focal zone, from the central A-line (number 50), which corresponds to the scatterers that reside around the target axis of symmetry where, as discussed in the previous sections, the lateral motion is minimized. Under these conditions, the decrease of the measured correlation coefficient with tissue strain is accounted for only by the axial decorrelation. As shown in Fig. 10, the measured correlation is predicted by the theory given by eqn (3) within 2 standard deviations of the correlation coefficient estimation error. The high variance is due to the noise in the RF signals that is not accounted for by
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0.3  0.5  0.7  0.9  1.1
Correlation coefficient vs. applied strain

0  1  2  3  4
Axial applied strain (%)

Fig. 11. Measured (—■—) and predicted (—) maximum correlation coefficients vs. applied strain. The measured correlation coefficient was obtained using cross-correlation technique applied to data segments from pre- and uniformly stretched postcompression RF A-lines. The segment size is 2.5 mm. This correlation is an average of 10 overlapping segments. The error bars represent 4 standard deviations. The correlation coefficients were measured using the A-lines close to one edge of the array transducer (y = 20 mm). The predicted maximum correlation coefficient was obtained using both eqns (3) and (12) (Lz = 0.38 mm).

0.3  0.5  0.7  0.9  1.1
Correlation coefficient vs. applied strain

0  1  2  3  4
Axial applied strain (%)

Fig. 12. Measured (—■—) and predicted (—) maximum correlation coefficients vs. applied strain. The measured correlation coefficient was obtained using cross-correlation technique applied to data segments from pre- and uniformly stretched postcompression RF A-lines. The segment size is 1.3 mm. This correlation is an average of 10 overlapping segments. The error bars represent 4 standard deviations. The correlation coefficients were measured using the center A-lines of the array (y = 0 mm). The predicted maximum correlation coefficient was obtained using eqn (3).

the theory of eqn (3). The predicted correlation coefficients are obtained using the following parameters: Lz = Lw = 0.08 mm, Z = 2.5 mm and f0 = 7.5 MHz.

Figure 11 shows the same results as Fig. 10, but when the A-line near the edge of the array transducer (number 99) is used to compute the correlation coefficients. In this case, both axial and lateral decorrelation account for the decrease of the correlation coefficient with the increase of tissue strain. The measured correlation in Fig. 11 is predicted by the theory within 2 standard deviations of the correlation coefficient estimation error [product of eqns. (3) and (12)]. This high variance is related to the sonographic noise not accounted for by the theory. The beam parameter Lw used in eqn (12) is set equal to 0.36 mm, as experimentally measured in a previous work using the same gel phantom (Kallel and Ophir 1997).

Figure 12 shows the same results as Fig. 10, but when a 1.3-mm window size is used to compute the cross-correlation function. Notice the high variance of the estimated correlation coefficient, which is due to the sonographic noise in the RF data. In this case, the model prediction is much less accurate compared to the results obtained when a larger window size is used (Figs. 10 and 11). We also notice a significant bias between the predicted and the measured correlation coefficient.

Illustration of the nonstationarity in the SNR: simulation results

In this section, we illustrate the nonstationarity of SNR in simulated elastograms using a finite-element (FE) commercial software (Linear stress, Algor, Inc., Pittsburgh, PA) and a 2-D ultrasound image formation model. The simulated tissue is a cube of 40 X 80 X 60 mm³. This cube is divided into 1000 brick elements for FE analysis. A Young’s modulus of 21 kPa with a Poisson’s ratio of 0.495 is used. A uniform displacement of 0.6 mm is applied in the axial direction and the tissue was set free in both the axial and lateral directions. Figure 13a, b shows the (a) axial and (b) lateral components of the displacement field in the acoustical scanning plane (y, z). Notice that these displacement components are 1-D functions of spatial position. As shown in Fig. 13b, the lateral tissue motion is close to zero around the center of the target (y = 0 mm).

The axial and lateral components of the displacement field are used to change the position of the tissue scatterers. Following the procedure described by Kallel and Bertrand (1996), pre- and postcompression RF images of the tissue are generated. For this simulation, a 7.5-MHz center frequency with 50% bandwidth and a 0.7-mm stationary lateral beamwidth are considered. The sampling frequency is 48 MHz in the axial direction and 0.3 mm/pixel in the lateral direction. The
Fig. 13. Mesh plot of the component of the displacement field in the acoustical scanning plane. (a) Axial displacement field. (b) Lateral displacement field. Due to the perfect applied slip boundary condition, the axial displacement is a function only of the axial position (1-D function) and the lateral displacement is a function only of the lateral position (1-D function).

tissue is modeled by a 2-D Gaussian distributed random field with a uniformly distributed scatterer density of 15 scatterers/wavelength. The strain image computed using a cross-correlation technique applied to the simulated RF images is shown in Fig. 14a. From this elastogram, the lateral nonstationarity of the SNR is not obvious. As shown by Fig. 14b, the difference in the $SNR_e$ from the center to the edges is small. This slow variation of $SNR_e$ with lateral position is also predicted using the strain filter generated for the same parameter values. This theoretical prediction is also shown in Fig. 9. It is, however, scaled here to overlap with the measured $SNR_e$ (the predicted $SNR_e$ are divided by a constant set empirically equal to 1.5). Observe that the theory accurately predicts the nonstationarity of the $SNR_e$ observed using simulated data. To exaggerate the nonstationarity of the $SNR_e$, the lateral tissue displacement is doubled while keeping the same axial displacement field. In practice, this is equivalent to the confinement of the tissue in the elevational direction for an incompressible material. Under these conditions, the tissue is said to be in a plane-strain state condition that is required to solve the inverse problem in elastography as proposed by Kallel and Bertrand (1996). Figure 15a, b shows the corresponding elastogram and a plot of the $SNR_e$ as a function of lateral position. Notice the drop of $SNR_e$ as we approach the transducer array edges. This drop is accurately predicted using the SF (Fig. 9).
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8

aA
$6
"4
3
2
1
0
-20
-10
0
10
20

Lateral position (mm)

Fig. 15. (a) Simulated elastogram obtained after confining the tissue in the elevational direction. In this case, the lateral motion is doubled. Notice that, as we approach the edges of the target, the noise increases. (b) Average SNR, as a function of lateral position. Notice the increase of the decay rate of the SNR, with lateral position. (-) measured SNR; (----) theoretically predicted SNR.

Illustration of the nonstationarity in the SNR: experimental results from phantom

To show experimentally the nonstationarity in an elastogram, we conducted the following experiment using the same homogeneous phantom previously described. The phantom is compressed using a circular plate in which the transducer array is inserted. In this experiment, the compressor is slightly smaller than the gel phantom. To reduce the motion of the tissue scatterers out of the acoustical scanning plane, the phantom is confined in the elevational direction between 2 rigid plates. The digitized data were collected from a 40 x 50 mm² ROI centered around the transmit focus and for an applied strain of 1%. Pre- and postcompression RF images were collected. Each image consisted of 100 A-lines.

Figure 16a, b shows the resulting elastogram and a plot of the SNR as a function of lateral position, averaged over a total depth of 10 mm in the focal zone at 20 mm from the transducer face. As shown by the elastogram, the variance increases from the center to the edges of the array transducer. Figure 16b shows a plot of the SNR as a function of the lateral position. As shown by Fig. 9, such variation of the SNR with lateral position is accurately predicted by the strain filter.

As we previously suggested, one way to reduce the
Fig. 17. (a) Elastogram obtained after confining the phantom in the lateral direction. This image is displayed at the same dynamic range as the elastogram of Fig. 13a. When compared to the elastogram of Fig. 13a, an important improvement in the SNR, in the elastogram of the laterally confined phantom is obtained. (b) Plot of the SNR, as a function of lateral position. (----) The phantom is confined in the elevational direction; (- - -) the phantom is confined in the lateral direction.

The nonstationarity of the SNR, with the lateral direction consists of confining the tissue in the lateral direction. However, this doubles the scatterer motion in the elevational direction (out-of-plane) and one might think, rightly, that this would increase the signal decorrelation. Fortunately, the ultrasound beam is wide in this direction and generally the compressor is on the axis of symmetry of the target where the elevational motion is small. In the current study, this is the case because a homogenous target is used. The resulting elastogram is shown in Fig. 17a. This elastogram is less noisy compared to the elastogram of Fig. 16a. Figure 17b shows the SNR, as a function of lateral position averaged over a total depth of 10 mm in the focal zone, computed using the elastograms of Fig. 16a (dotted line) and 17a (solid line). These plots clearly show that confining the tissue in the lateral direction improves the SNR, and reduces its nonstationarity with lateral position relative to the center of the linear array transducer.

CONCLUSIONS

Understanding the noise sources in elastography is the key to further improvement of the quality of elastograms. The strain filter is a useful graphical and analytical tool developed to relate the noise sources to the signal-processing parameters and data-acquisition strategies. In this paper, we have used this tool to examine the nonstationary variation of the elastographic SNR, with lateral motion of the tissue scatterers. Theoretical, simulation and experimental results illustrate that the SNR, decreases as the position of the A-line signals varies from the center of the linear array transducer to its edges. The rate at which the value of the SNR, decreases depends on both the tissue strain and the ultrasound beamwidth.

The strain filter predicts the reduction in the elastogram quality due to lateral signal decorrelation. The 3-D plot of the strain filter clearly presents the performance of the cross-correlation-based strain estimator in elastography. Tradeoffs between the use of different boundary conditions (lateral tissue confinement) used to improve elastogram quality is also analyzed using the strain filter. The trend in the variation of the SNR, with lateral displacement, obtained using finite-element simulations and experimental results on the uniformly elastic phantom, follows the theoretical curve predicted from the strain filter.

Both the computer simulation with finite-element analysis and the phantom experiment corroborate the nonstationary behavior predicted by the theory using the strain filter. Confinement of tissue in the lateral direction reduces lateral tissue motion and significantly improves the elastogram quality, as demonstrated by the theoretical and experimental results. The nonstationary variation in the elastographic SNR, is minimized by lateral confinement. Lateral confinement of tissue can, therefore, help in the detection of low contrast lesions near the edges of the array transducer that may otherwise be lost in the elevated background noise due to lateral signal decorrelation. However, in more general inhomogeneous and anisotropic tissue situations, an alternative way to reduce both lateral and out-of-plane decorrelation is to confine the tissue in the elevational direction and use an algorithm to track speckle motion in both axial and lateral directions (Chaturvedi et al. 1997).
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