POWER SPECTRAL STRAIN ESTIMATORS IN ELASTOGRAPHY

E. E. KONOFAGOU,*†‡ T. VARGHESE,* J. OPHIR*†‡ and S. K. ALAM*

*Ultrasonics Laboratory, Department of Radiology, The University of Texas Medical School, Houston, TX 77030 USA; and †Biomedical Engineering Program and ‡Department of Electrical Engineering, University of Houston, Houston, TX 77004 USA

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Abstract—Elastography can produce quality strain images in vitro and in vivo. Standard elastography uses a coherent cross-correlation technique to estimate tissue displacement and tissue strain using a subsequent gradient operator. Although coherent estimation methods generally have the advantage of being highly accurate and precise, even relatively small undesired motions are likely to cause enough signal decorrelation to produce significant degradation of the elastogram. For elastography to become more universally practical in such applications as hand-held, intravascular and abdominal imaging, the limitations associated with coherent strain estimation methods that require tissue and system stability, must be overcome. In this paper, we propose the use of a spectral-shift method that uses a centroid shift estimate to measure local strain directly. Furthermore, we also show theoretically that a spectral bandwidth method can also provide a direct strain estimation. We demonstrate that strain estimation using the spectral-shift technique is moderately less precise, but far more robust than the cross-correlation method. A theoretical analysis, simulations and experimental results are used to illustrate the properties associated with this method. © 1999 World Federation for Ultrasound in Medicine & Biology.

Key Words: Bandwidth, Centroid, Cross-correlation, Elastogram, Elastography, Imaging, Spectral shift, Strain, Strain Filter, Ultrasound.

INTRODUCTION

Imaging of elastic parameters of soft tissue has developed into a new tool for diagnosis of disease (Wilson and Robinson 1982; Krouskop et al. 1987; Lerner et al. 1990; Yamakoshi et al. 1990; Parker et al. 1990; Ophir et al. 1991, 1996, 1997; O’Donnell et al. 1991; Talhami et al. 1994; Gao et al. 1996; Garra et al. 1997; deKorte et al. 1997). Current estimators of tissue motion include a time-domain cross-correlation–based speckle tracking algorithm (Ophir et al. 1991; Céspedes et al. 1993), and a Fourier-based speckle phase-tracking technique (O’Donnell et al. 1991). These techniques are coherent estimation techniques (i.e., they are sensitive to phase variations). The coherent estimation techniques generally have the advantage of being highly precise. Strain Filter (SF) analysis has shown, however, that they are not very robust in the presence of even a moderate amount of decorrelation between the pre- and postcompression sig-

nals (Varghese and Ophir 1997; Kallel and Ophir 1997a; Varghese et al. 1998). The term “robustness” has been used in statistical analysis to denote the good performance of statistical tests, (i.e., the homogeneity of the variance calculation) even if the data deviate from theoretical requirements (Zar 1984). By equivalence in elastography, “robustness” denotes the consistently good performance of the estimator even at high decorrelation and random noise (i.e., keeping the variance of estimation at a relatively constant and low level at a large range of noise levels). Throughout this paper, decorrelation is defined as the loss of full correlation between the pre- and postcompressed windowed signal segments. Therefore, decorrelation may be encountered due to many sources, such as intrawindow axial motion (Céspedes 1993; Varghese et al. 1998), undesired lateral or eleva-
tional motion (Kallel and Ophir 1997a), jitter (i.e., any cause of misregistration between the pre- and postcompressed A-line segments), unstable mechanical setup, etc. The main idea in this study was to introduce a new estimator that is more immune to decorrelation than other estimators.

The tissue strain estimator presented in this paper is
a spectral estimator that estimates strain directly. Because the proposed estimator uses the power spectrum, it is incoherent (i.e., it does not use the phase of the signal). Previously reported incoherent methods include optical flow speckle tracking (Bertrand et al. 1989), envelope cross-correlation (Varghese and Ophir 1998a), and spectral chirp z-transform techniques (Talhami et al. 1994). Generally, incoherent methods may be less precise, but significantly more robust. For example, we have demonstrated this property for the case of time-delay estimation using the envelope of echo signals (Konofagou et al. 1997b; Varghese and Ophir 1998a). This may be a significant advantage where elastography is to be practiced in situations involving 1. undesired scanning motion, such as the case of using an unstable hand-held transducer (Bamber and Bush 1996) and/or 2. undesired tissue motion, such as abdominal or intravascular elastography (Talhami et al. 1994; Shapo et al. 1996; Soualmi and Bertrand 1996; DeKorte et al. 1997). This property of the estimator is demonstrated later in this paper in Simulation Results, through testing of its immunity to noise caused by jitter.

The main idea behind a spectral-strain approach is based on the Fourier scaling property, which implies that a compression or expansion of the time-domain signal should lead to an expansion or compression of its power spectrum, respectively. One of the most well-known and thoroughly studied spectral-motion effects is the Doppler shift, which typically links the frequency shift to the scatterer velocity between emissions. Velocities toward the transducer result in a positive frequency shift, and the opposite is true for scatterers that move away from the transducer. However, because the scatterers within a given resolution length do not move at the same velocities, a spectrum of Doppler frequencies is observed. Therefore, initially in ultrasound, the methods of velocity estimation for the measurement of blood flow mainly operated in the frequency domain, otherwise known as spectrum analysis techniques. de Jong et al. (1975) and Gerzberg and Meindl (1980) measured the mean velocity of scatterers across the vessel lumen (indicative of the volumetric flow rate) by estimating the mean frequency of the power spectrum. Despite the success of these techniques even in in vivo vessels, detection of the Doppler frequency shift, which is typically on the order of 1 kHz, is not possible for pulsed instruments because the downshift in frequency due to attenuation (on the order of 10–100 kHz) is expected to dominate over the Doppler shift (Jensen 1996). Because, in elastography, the pre- and postcompressed segments are approximately at identical depths, the attenuation effect on the two spectra is assumed to be identical and cancelled out when the two spectra are compared.

Strain estimation using spectral methods depends on the subsequent change in the scatterer statistics. Spectral methods typically link one or more signal parameters to the change in mean scatterer spacing (Fellingham and Sommer 1984; Kuc et al. 1986; Landini and Verrazini 1990; Varghese and Donohue 1993, 1994, 1995; Talhami et al. 1994; Varghese 1995). The approach proposed by Talhami et al. (1994) relates the relative change in the mean scatterer spacing to the strain incurred during a cardiac cycle. This method assumes the presence of underlying scatterer periodicities. Despite the fact that this has also been demonstrated to work in in vivo intravascular applications, the main assumptions of regular spacing or periodicities may not hold for most tissues. In contrast, as shown in Theory, the spectral methods mentioned in this paper make no assumptions regarding the composition of the tissue scatterers.

Typically, in elastography, time-domain techniques are used that involve the computation of the time-delay to estimate the displacement following an applied compression, and the estimation of strain by applying gradient operations on the previously obtained time-delay estimates. As mentioned earlier, an important advantage associated with these spectral methods, as well as other estimators, such as the adaptive stretching estimator (Alam et al. 1998), is that they can be used to estimate strain directly (i.e., without involving the use of noisy gradient operators). In the latter case, the gradient operation introduces additional amplification of the noise into the strain estimation process; thus, degrading the strain estimates (Kallel and Ophir 1997b). Furthermore, similar to the adaptive stretching estimator, only one estimation window is needed to estimate for both the magnitude and the sign of the strain.

As shown later in Theory, spectral estimators can be divided into two main groups: 1. the spectral-shift methods, and 2. the spectral-bandwidth methods. Despite the fact that we develop expressions that show direct strain estimation in both cases, in this paper we focus primarily on a spectral-shift method; we estimate the relative shift in the spectral centroid caused by compressive or tensile tissue strain (Varghese et al. 1999). Therefore, throughout this paper this new estimator is referred to as the centroid strain estimator, centroid estimator or centroid method. Current investigations that deal with the development of alternative shift estimators, as well as bandwidth estimators, will not be reported in this study.

The spectral centroid has been widely used in estimating the Doppler shift (Gerzberg and Meindl 1980; Jensen 1996), attenuation (Fink et al. 1983; Flax et al. 1983) and backscattering (Bahr et al. 1985). The theory underlying the use of centroid strain estimators is presented in the next section. One-dimensional (1-D), instead of two-dimensional (2-D), motion simula-
tions are used to more accurately study the performance of the estimator, \( i.e., \) independent of the effect of signal decorrelation in 2-D that complicates the measurements \( (Kallel \ and \ Ophir \ 1997a; \ Konofagou \ and \ Ophir \ 1998) \). Simulation results in 1-D illustrate the insensitivity of the centroid strain estimator to signal decorrelation effects. It is important to note that, as mentioned earlier, decorrelation can be due to several sources. In this paper, we consider solely the axial decorrelation effect in this 1-D model. We thus assume that the robustness demonstrated by the spectral estimator vis-\( \text{à}-\text{vis} \) this effect is a more general property that can be further applied at other decorrelation scenarios. For example, it is shown in the simulation results how the spectral method is, indeed, more immune to jitter, another source of decorrelation. The experimental results demonstrate that the spectral method are relatively immune to 3-D motion decorrelation as well. The elastograms obtained using these simulations, as well as phantom experiments, illustrate the robustness of the spectral centroid strain estimator. The properties of the new estimator are discussed and summarized in the Conclusion.

**THEORY**

In this section, we show analytically that, for Gaussian echo spectra, the relative spectral shift is a direct measure of tissue strain. We also show that the relative bandwidth variation can also be used as a direct strain estimator.

**Signal and noise model**

The pre- and postcompression echo signals are given as follows \( (Varghese \ et \ al. \ 1998) \):

\[
\begin{align*}
r_1(z) &= h(z) \ast e(z) + n_1(z) \\
r_2(z) &= h(z) \ast e(az) + n_2(z),
\end{align*}
\]  

where \( z \) is a spatial variable, \( r_1(z) \) and \( r_2(z) \) are the received RF signals before and after compression, respectively, \( h(z) \) is the impulse response of the ultrasound system or point-spread function (PSF), \( e(z) \) is the scattering function, \( n_1(z) \) and \( n_2(z) \) are independent zero-mean white noise sources and \( a \) is the compression coefficient \( (\text{or, strain factor}) \) linked to strain \( s \) through

\[
a = \frac{1}{1 - s} \approx 1 + s.
\]

The approximation holds for \( s \ll 1 \), where the strain \( s \) for a 1-D homogeneous target is typically defined in mechanics by

\[
s = \frac{L_0 - L}{L_0},
\]

where \( L_0 \) and \( L \) are the pre- and postcompressed axial dimensions of the target. From eqn (4), the reader should note that positive strain denotes compression \( (a > 1) \) and negative strain denotes tension \( (a < 1) \). Throughout this paper, the subscripts 1 and 2 denote pre- and post-compression parameters, respectively.

Assuming that \( h(z) \) and \( e(z) \) in eqn (1) and (2) can be described by their autocorrelation functions that may be modeled by modulated Gaussian functions \( (Wagner \ et \ al. \ 1983; \ Lizzi \ et \ al. \ 1987; \ Insana \ et \ al. \ 1990; \ Bilgen \ and \ Insana \ 1996; \ Varghese \ et \ al. \ 1998) \), we obtain:

\[
h(z) = \frac{1}{\sqrt{2\pi L_h}} \exp(-z^2/2L_h^2)\sin(k_hz)
\]

and

\[
e(z) = \frac{1}{\sqrt{2\pi L_e}} \exp(-z^2/2L_e^2)\cos(k_hz),
\]

where \( L_h \) and \( L_e \) are the resolution lengths of the PSF and of the scattering function, respectively, \( k_h \) is the central spatial frequency of the PSF, and \( k_z \) is the central spatial frequency of the scattering function.

The one-sided power spectra of the pre- and post-compression RF signals \( (\text{positive frequencies}) \) are given, respectively, by:

\[
R_1(k) = \frac{1}{4} \exp\left(-\frac{1}{2} \left[ (k - k_h)^2L_h^2 + (k - k_z)^2L_e^2 \right] \right) + N_1(k)
\]

\[
R_2(k) = \frac{1}{4a} \exp\left(-\frac{1}{2} \left[ (k - a k_h)^2L_h^2 + (k - a k_z)^2L_e^2 \right] \right) + N_2(k)
\]

where \( N_1(k) \) and \( N_2(k) \) are independent power spectra of zero mean white noise processes \( (i.e., \)

\[
\langle N_i(k) \rangle = \langle N_2(k) \rangle = 0).
\]

A brief observation of eqns (7) and (8) reveals a centroid shift in the scattering spectrum resulting from the com-
pression. In other words, if \( f_{c1} \) and \( f_{c2} \) are the center frequencies of the scattering spectrum before and after compression, respectively, and, assuming that the speed of sound in the tissue \( c \) remains constant, from eqns (7) and (8) we have:

\[
f_{c2} - f_{c1} = \frac{c}{2\pi} \left( ak_e - k_e \right) = (a - 1) f_{c1},
\]

(9)
or, from eqn (3):

\[
f_{c2} - f_{c1} \approx s.
\]

(10)

So, the relative centroid shift in the scattering function spectrum constitutes a direct strain estimator. A similar result is found later (eqn 16) using the spectrum of the received signal. In the section below, we use eqn (10) as a guide in the formulation of the new estimator.

Effect of strain on the spectrum of the received signal

The centroid of the power spectrum of the received signal is defined as follows (Bracewell 1978):

\[
f_c = \frac{1}{2\pi} \int_{-\infty}^{\infty} kR_i(k)dk
\]

(11)

The centroid estimate for the precompression power spectrum (derived in Appendix A) is given by:

\[
f_{c1} = \frac{c}{2\pi} \frac{k_eL_e^2 + k_hL_h^2}{L_h^2 + L_e^2}.
\]

(12)

In a similar manner, we can derive the expression for the centroid of the postcompression power spectrum by replacing \( k_e \) and \( L_e \) in eqn (9) by their corresponding parameters in the postcompression power spectrum (i.e., \( ak_e \) and \( L_e/a \)) (as indicated from eqn 7) to obtain:

\[
f_{c2} = \frac{c}{2\pi} \frac{k_eL_e^2}{a^2} \frac{L_e + L_h^2}{L_h^2 + L_e^2}.
\]

(13)

Note that the PSF parameters remain unchanged. Because both centroids depend on the center frequencies and bandwidths of the scattering function and the PSF, and by consulting eqn (10), we normalize this effect by using the following ratio as a candidate strain estimator:

\[
\frac{f_{c2} - f_{c1}}{f_{c1}} = \frac{k_ea^2L_e^2 + ak_eL_e^2}{a^2L_h^2 + L_e^2} = \frac{k_eL_e^2 + k_eL_e^2}{L_h^2 + L_e^2}.
\]

(14)

The parameters \( L_e \) and \( L_h \) are related to \( B_e \) and \( B_h \), the equivalent noise spectral bandwidths (Bendat and Piersol 1986) for the scattering and PSF spectra, through:

\[
B_h = \frac{1}{2\sqrt{\pi} L_h}
\]

(15)

and

\[
B_e = \frac{1}{2\sqrt{\pi} L_e},
\]

(16)

respectively. However, the PSF bandwidth is typically much smaller than the bandwidth of the scattering function (Bilgen and Insana 1996; 1997; Bertrand et al. 1989), i.e., \( B_e \gg B_h \); thereby, \( L_e \ll L_h \) and, therefore, \( a^2L_h^2 \approx L_h^2 \). After cancellation of common terms in the numerator and denominator of eq. (14), we obtain:

\[
\frac{f_{c2} - f_{c1}}{f_{c1}} \approx \frac{(a - 1) k_eL_e^2}{k_eL_h^2 + k_eL_e^2} = \frac{(a - 1)}{L_h^2 + L_e^2}.
\]

(17)

or, from the small strain approximation case of eqn (3) (i.e., in mathematical terms, for strains less than 10%),

\[
\frac{f_{c2} - f_{c1}}{f_{c1}} \approx As,
\]

(18)

where \( A \) is given by:

\[
A = \frac{1}{k_eL_h^2 + 1}
\]

(19)
or,

\[
A \approx \frac{k_eB_h^2}{k_eB_e^2}.
\]

(20)
Inspection of eqn (18) leads to the following interesting observations:

- The relative spectral centroid shift can be used as a direct strain estimator. We can also observe a direct analogy between the classic definition of strain, eqn (4), and the estimator of eqn (18), which establishes this method as a simple and straightforward way of estimating the strain.

- When the strain is positive (or compressive), a frequency upshift occurs (i.e., \( f_{c2} - f_{c1} > 0 \)). Conversely, a tensile (or negative) strain results in a frequency downshift (i.e., \( f_{c2} - f_{c1} < 0 \)). Therefore, the estimator of eqn (18) provides directly not only the magnitude of the strain, but also its sign.

- Because constant \( A \) is independent of the strain, it will introduce a uniform bias on the resulting elastogram. This should not affect the resulting elastogram because the latter depicts relative values of strain. The reader should note that the effect of local bandwidth variations is ignored.

- The scattering spectrum must be a bandpass and band-limited spectrum to estimate the strain using the centroid estimator. Otherwise, if \( B_0 \) is infinite and/or if \( k_c \) is zero, constant \( A \) in eqn (18) will always be zero, regardless the strain.

- For relatively larger strains, using the more general form of eqn (3), eqn (18) becomes:

\[
\frac{f_{c2} - f_{c1}}{f_{c1}} \equiv A \frac{s}{1 - s},
\]

or, by solving for the strain,

\[
s \equiv \frac{f_{c2} - f_{c1}}{(A - 1)f_{c1} + f_{c2}},
\]

which is a less straightforward, but still a direct way of estimating higher strains.

- Equation (18) is reminiscent of the well-known Doppler effect, in which scatterer velocity causes a shift in center frequency. According to eqn (18), in the case of strain, a similar effect also occurs. Also, Newhouse and Amir (1983) among others, have shown how, in broadband Doppler, the bandwidth of the resulting spectrum also changes with velocity and that the output RF Doppler spectrum is a frequency-shifted and compressed (or stretched) replica of the transmitted one. Similarly, in the case of strain measurement, we prove in Appendix B, that the following expression provides a direct estimation of strain:

\[
\frac{B_2 - B_1}{B_1} \equiv ks,
\]

where \( B_2 \) and \( B_1 \) are the post- and precompression bandwidths, respectively and \( k \) is a constant. So, strain, like velocity, introduces these two coupled effects of centroid shift and bandwidth variation in the power spectrum.

In Appendix B, we also show that spectral broadening (i.e., in the case of signal compression) or contraction (i.e., in the case of signal tension) can introduce a bias in the measurement. A general expression is derived, linking the centroids \( f_{c1} \) and \( f_{c2} \) (before and after compression, respectively), the pure frequency shift (i.e., in case the pre- and postcompression spectra are identical, only centered at different frequencies separated by a shift), \( \Delta f \), and a bias term \( \beta \) denoting the spectral broadening (or, compression) due to strain \( s \):

\[
f_{c2} - f_{c1} = \Delta f + \beta(s),
\]

where

\[
\beta(s) = B_2(s) - B_1.
\]

To estimate the strain without the bias associated with spectral broadening, the following equation can be used, that results from eqns (18), (23), (24) and (25):

\[
\Delta f \equiv A f_{c1} s - k B_1 s
\]

and solving for strain, the unbiased estimator is given by:

\[
s \equiv \frac{\Delta f}{A f_{c1} - k B_1}.
\]

However, eqn (27) requires a bandwidth estimation and, because the bandwidth estimator is not part of this study, we use eqn (18) as the strain estimator and show a bias with simulations, which is partly due to the previously described bias due to spectral broadening.

**SIMULATIONS**

Simulation results using a 1-D scattering model are shown in this section to illustrate the performance of the centroid strain estimator. Strain estimation using the centroid is also compared to the standard cross-correlation-based algorithm.

**Methods**

Monte-Carlo simulations in MATLAB (Mathworks, Inc., Natick, MA) are used to generate pre- and postcompression RF signals for a 30-mm target segment and sampled at 48 MHz. The speed of sound in tissue was assumed to be constant at 1540 m/s. The PSF was simulated using a Gaussian modulated cosine pulse with a wave number =
20.4 mm$^{-1}$ (5-MHz center frequency, 50% bandwidth), and a 0.2218-mm standard deviation, unless stated otherwise. The scattering function consisted of randomly distributed point scatterers following a uniform distribution with density of 40 scatterers/pulse-width to simulate Gaussian statistics. We assume that the uniformly distributed scatterers are of a sufficient number to generate an echo signal with circular Gaussian statistics (Wagner et al. 1986; Insana et al. 1986, 1990). The PSF was convolved with the scattering function to obtain the precompression RF signal. The postcompression signals were generated after applying a uniform compression of the point scatterers (Césedes 1993), and convolving the compressed point scatterers with the original PSF.

Spectral strain estimation (following eqns 18 and 11) was performed using pre- and postcompressed power spectra of windowed RF signals. The signal length equaled 4 mm and the size of the postcompression window was changed with strain to assure that the same tissue information was incorporated in both the pre- and postcompressed windows (Césedes 1993). The length of the data segment incurred the usual trade-off in spectral estimation. A larger window length improved the spectrum as long as the data were stationary.

The power spectral calculation of the pre- and postcompressed RF segments was performed using a 25-point frequency smoothing window, unless otherwise stated. The 25-point frequency smoothing window represents only 0.6% of the entire FFT and is a relatively small window. Frequency smoothing is similar to using a moving-average window; however, the averaging is performed on the complex spectrum to obtain an estimate of the power spectrum. Frequency smoothing allowed the use of a single pair of A-lines, similar to the strain estimation performed using the cross-correlation-based strain estimator. We use a 1024 point chirp Z-transform to compute the spectrum that would correspond to a 4096-point FFT (because we use only one half of the spectrum, and only the region with a sufficient signal). The mean and standard deviation of the strain estimates were obtained by processing pre- and postcompression A-lines with a total length of 30 mm. The corresponding SNR$_e$ (ratio of the mean of the estimated strain to its standard deviation) values were obtained using Monte Carlo simulations in MATLAB with 25 independent realizations for each strain value. The simulated Strain Filters (Varghese and Ophir 1997a) were obtained by plotting the SNR$_e$ estimates for the whole range of the applied tissue strains. The Strain Filter typically addresses the limitations of the ultrasound system (such as time-bandwidth product, center frequency and sonographic SNR), as well as the signal-processing algorithms used to process the signals through the introduction of constraints in the attainable elastographic SNR, resolution, sensitivity and strain dynamic range (Varghese and Ophir 1997a).

The robustness of the strain estimators was also evaluated by introducing jitter errors in the scatterer positions before generating the postcompression signals. The jitter in the scatterer positions followed a normal distribution that varied randomly from zero to the maximum value of the jitter introduced. For the larger jitter values, the scatterers could move out of the window of estimation. The strain estimation accuracy and precision for the coherent estimators were expected to deteriorate under these conditions because they depend on the relative motion of the scatterers themselves with compression. However, the centroid estimator, being incoherent, was expected to show strain estimation with a reasonable SNR$_e$, even at high jitter levels.

Results

An example of the frequency shift on simulated spectra of entire A-lines (40 mm in length) is shown in Fig. 1 for the case of 1% applied strain. Comparisons of the strain estimators using the coherent cross-correlation and centroid-based algorithms are presented in Figs. 2 and 3 for the 1-D simulations. The mean strain estimates and their standard deviations are presented in Fig. 2 and the respective simulated Strain Filters are presented in Fig. 3. The results in Fig. 2 illustrate that the strain estimates from both estimators follow the theoretical curve (straight solid line at 45°) for strains less than 5%, where the cross-correlation strain estimator begins to level off, and at 8% where the centroid strain estimator crosses the theoretical curve. Both estimators are biased, with a small overestimation of the strain seen for strains lower than 5%. For larger strains, the centroid estimator underestimates the actual strain values, by introducing a larger bias in the estimated strain value. This bias in the strain estimation for the centroid estimator is, at least partly, due to the bandwidth broadening, as discussed in Theory. The bias in the cross-correlation-based strain estimator is due to the errors associated with tissue compression that corrupt the time-delay estimates. These bias errors can be reduced by temporally stretching the postcompression data, as illustrated by Varghese and Ophir (1998). Overall, when compared to the standard elastographic coherent estimator, the centroid strain estimator provides a biased, but more robust, strain estimate.

In Fig. 3, the simulation Strain Filters (Varghese and Ophir 1997a) for the two algorithms illustrate the noise performance of the estimators. Note that the coherent cross-correlation strain estimator provides accurate and precise strain estimates for strains less than 2% and the variance increases rapidly beyond this strain value; however, for larger strains, the performance deteriorates significantly. On the other hand, the centroid strain estimator, not being sensitive to phase, provides a
robust strain estimate, even at very large strains close to 30%. The SF for the centroid strain estimator indicates a reasonable SNR for low tissue strains, as well as an increase in the SNR observed for larger strains where the cross-correlation strain estimator is limited by signal decorrelation errors. Due to its lower precision, the centroid estimator works best at higher strains, where the shift is greater and, therefore, the signal-to-noise ratio (assuming the variance remains constant) increases, as shown in Fig. 3.

Next, we investigated the sensitivity of the cross-correlation and spectral strain estimator to variations in the scatterer positions caused due to axial jitter. The results are illustrated in Figs. 4 and 5 for the cross-correlation and centroid strain estimators, respectively. Note that the coherent cross-correlation-based strain estimator is more susceptible to jitter than the centroid strain estimator. The noise performance of the cross-correlation estimator drops by about 50%, with an increase in the maximum value of the jitter by 50 ns. However, in the case of the centroid strain estimator, the noise performance remains at the same high level, even at jitter magnitudes of 100 ns. The simulations, therefore, show the robustness of the spectral centroid strain estimator to large applied strains and increased jitter, errors that are most likely to be encountered in hand-held, intravascular or abdominal elastography. The two following sections compare elastograms obtained with these two estimators in the case of a 2-D finite-element simulation undergoing 1-D motion and an experimental phantom.

ELASTOGRAMS USING SIMULATED DATA

Methods

After testing the properties of the new estimator, elastograms were generated for a simulated single inclusion phantom under uniform compression. For the calculation of the displacements, we used a finite element analysis (FEA) commercial software (ALGOR, Inc., Pittsburgh, PA). The simulated totally compressible and isotropic phantom contained a single inclusion three times stiffer than the homogeneous background (background modulus = 21 kPa). All nodes were constrained to move solely in the axial direction, thereby avoiding decorrelation in other directions. This motion model was, therefore, considered 1-D. The scatter-
Fig. 2. Comparison of the strain estimation using cross-correlation vs. the centroid strain estimator. The strain estimates are obtained using a window length ($Z$) of 4 mm with a 50% overlap between data segments. The dotted and solid error bars correspond to the centroid and cross-correlation estimates, respectively. Note that in the case of the centroid estimator, the estimates are within one standard deviation of the true values, unlike the cross-correlation estimates.

ers were normally distributed. The ultrasonic parameters were as follows: center frequency 5 MHz, 50% −6 dB bandwidth, 100 A-lines, and no attenuation or beam width effects.

The strain estimation noise performance of the centroid estimator was compared to that of the standard elastographic cross-correlation–based strain estimator without motion compensation, (i.e., global stretching) (Céspedes et al. 1993; Varghese and Ophir 1997b; Alam and Ophir 1997). As explained in the introduction, mo-

Fig. 3. The simulated Strain Filters comparing the noise performance between the RF cross-correlation and spectral strain estimators.
Fig. 4. The deterioration in the SF performance for the coherent cross-correlation–based strain estimator with (a) 0 ns (solid error bars); (b) 50 ns (--- error bars); and (c) 100 ns (solid error bars) axial jitter. Note that the noise performance of the strain estimator deteriorates rapidly with the increase in the jitter, as illustrated from the curves with 50 and 100 ns jitter.

Fig. 5. The Strain Filters for the spectral strain estimator using the centroid shift with (a) 0 ns (--- error bars); (b) 50 ns (--- error bars); and (c) 100 ns (solid error bars) axial jitter. Note that the strain estimation noise performance does not deteriorate significantly with axial jitter up to 100 ns.

Results
Figure 6 presents the elastograms obtained using both these methods, along with the ideal elastogram (i.e.,
true strain image) for three different applied compressions. Note that, at the low strain value of 1% (Fig. 6i), the elastogram generated using the cross-correlation algorithm provides the closest correspondence to the ideal elastogram. On the other hand, for larger applied strains (5% and 10%, Fig. 6ii and iii), the cross-correlation algorithm fails to accurately estimate tissue strain due to the increased signal decorrelation errors (Fig. 6b, ii and iii, respectively). In fact, at 5% applied compression, part of the inclusion is still visible, being 3 times stiffer than the background, and thus undergoing lower strain, allowing it to be depicted with a good signal-to-noise ratio (Konofagou et al. 1997a). On the other hand, the elastograms generated using the spectral centroid method at 5% and 10% compression (Fig. 6c, ii and iii, respectively) illustrates the robustness associated with the centroid method.

In the next section, we present elastograms obtained using an elastographic experimental phantom. The experimental results provide a complete 3-D situation where axial, lateral and elevational signal decorrelation are present, unlike the 1-D situation illustrated in this section.
ELASTOGRAMS USING EXPERIMENTAL DATA

Methods

The ultrasound system used for acquiring the data was a Diasonics Spectra II real-time scanner (Diasonics Inc., Santa Clara, CA) operating with dynamic receive focusing and a single transmit focus centered at a depth of 3 cm. The transducer used was a 5-MHz linear array (40 mm) with a 60% fractional bandwidth. The digitizer used is a 8-bit digitizer (LeCroy Corp., Spring Valley, NY) with a sampling rate of 48 MHz. The digitized data was collected from a 40 × 50 mm ROI consisting of 100 A-lines (starting at a depth of 5 mm under the transducer) centered around the transmit focus. The system also included a motion control system, and a compression device. A personal computer controlled the operation of the entire system. A complete description of the elastography system has been given by Céspedes (1993).

A gelatin phantom\(^1\) (90 × 90 × 90 mm\(^3\)) containing a cylindrical inclusion with a 20-mm diameter, positioned at the center of the phantom and three times stiffer than the background, was used to compare the performance of the strain estimators (Hall et al. 1997). The phantom contained scatterers (graphite flakes), and was used to obtain RF scans before and after compression. A large compressor was used to simulate uniform stress conditions in the phantom. The phantom was lubricated on the top and bottom surfaces with corn oil to simulate slip boundary conditions, and was free on both lateral and elevational sides.

Results

Comparison of the estimation performance using coherent cross-correlation and centroid strain estimators is illustrated qualitatively using elastograms obtained at both low (0.5%) and high (3%) applied strains in Fig. 7. Note that coherent strain estimation provides the elastogram with the highest SNR\(_c\) for the low compression of 0.5% (Fig. 7i), when compared to the centroid method. However, for the large applied compression of 3% (Fig. 7ii), the coherent strain estimator fails completely when compared to the spectral centroid method, which pro-

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\(^1\) The elastographic phantom was supplied courtesy of Dr. Timothy Hall (Hall et al. 1997).
duces a reasonable elastogram. In addition, averaging several elastograms obtained from independent pre- and postcompression data can be used to further improve the elastograms for the centroid method. However, averaging is not useful for the coherent cross-correlation strain estimator in this case because the RF signals are completely decorrelated (Fig. 7a, ii).

Two major differences can be observed between the 1-D simulation results and the experimental results: 1. The mechanical artifacts for the 1-D simulation elastograms are along the top and bottom axes of the inclusion, when compared to the more complicated artifacts observed for the 3-D case, and 2. the cross-correlation algorithm fails at relatively lower applied strains (3% instead of 5% or 10%) than in the simulation results, due to lateral and elevational decorrelating motions involved. The centroid method yields similar results in both cases of low and high applied strains, demonstrating its robustness in a 3-D scenario as well. These results are comparable to what has been obtained with the iterative correction method (Konofagou and Ophir 1998; Konofagou et al. 1998) and may indicate, despite its lower precision, a more computationally efficient, as well as robust, method of estimating axial strain.

CONCLUSIONS

The new concept described in this paper is based on the direct estimation of tissue strain from the relative frequency shift in the power spectrum. The estimator hereby presented, namely the centroid-shift estimator, measures the shift by calculating the relative centroid shift resulting from the applied compression. This estimator has two major characteristics: it is 1. direct and 2. spectral (i.e., operates in the frequency domain). The direct strain estimation assures that no noise is added through the use of gradient operators, as is the case in time-delay based elastographic techniques. The spectral characteristic makes this method more robust because it is phase-independent and, therefore, suffers less from motion-induced decorrelation noise. Another estimator that is theoretically shown to provide a direct measure of strain is the resulting relative change of bandwidth in the power spectrum. The bandwidth parameter can also be used to eliminate the bias corrupting the centroid estimator.

To study the performance of the centroid estimator, we used a 2-D simulation model undergoing 1-D motion that allowed the scatterers to move solely in the axial direction. Preliminary results obtained with these simulations are used to demonstrate the robustness of the proposed method. Strain estimates as high as 10% are produced at a reasonably high signal-to-noise ratio, and the standard cross-correlation–based elastographic method practically failed beyond the levels of 2%. The 1-D motion example was preferred (to 2-D or 3-D simulations), so that the performance of the method could be characterized independent of noise due to 2-D or 3-D motion. If the 2-D or 3-D scenario were used, the estimators would fail at lower strains, but the spectral centroid estimator, not being sensitive to phase changes, would still be more robust than the RF cross-correlation estimators. In fact, phantom experiments were used to show that the centroid estimator could generate quality elastograms at applied strains as high as 3%, and the cross-correlation–based elastograms are extremely noisy. Furthermore, simulation results showed that the spectral centroid-shift method provided a jitter-insensitive method for estimating the strain. Therefore, spectral strain estimation may be particularly useful for obtaining good elastograms in noisy jitter environments produced by unpredictable tissue and/or system motion. This may constitute a major advance because elastography might be practiced using the same clinical guidelines as those employed by ultrasound (i.e., using a hand-held transducer). In addition, the jitter resistance of the centroid estimator could make it suitable for use in intravascular elastography in vivo, a task that has been demonstrated to be difficult using cross-correlation techniques (deKorte 1999). Future investigations will involve theoretical study of the performance of spectral (i.e., frequency shift and bandwidth) strain estimators, as well as experimental verification of their jitter insensitivity.

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**APPENDIX A**

*Derivation of the centroid estimator*

The centroid or center frequency estimate is given by eqn (8); we first evaluate the numerator in eqn (8) for the precompression signal of eqn (7):

\[
\int_{-\infty}^{\infty} k R_c(k) dk = \frac{1}{4} \int_{-\infty}^{\infty} k \exp \left( -\frac{1}{2} \left[ (k - k_1)^2 L_1^2 + (k - k_2)^2 L_2^2 \right] \right) dk.
\]

(A-1)

Simplification of eqn (A-1) by expanding the terms inside the exponential and completing the square leads to:

\[
\int_{-\infty}^{\infty} k R_c(k) dk = A \int_{-\infty}^{\infty} k \exp \left( -\frac{1}{2} (L_1^2 + L_2^2) \left[ (k - \frac{k_1 L_1^2 + k_2 L_2^2}{L_1^2 + L_2^2})^2 \right] \right) dk.
\]

(A-2)

where

\[
A = \frac{1}{4} \exp \left( -\frac{1}{2} \left[ k_1^2 L_1^2 + k_2^2 L_2^2 - (k_1 L_1^2 + k_2 L_2^2)^2/(L_1^2 + L_2^2) \right] \right).
\]

(A-3)

The integral in eqn (A-2) is in the form of a first moment of a Gaussian with the mean and standard deviation given by:

\[
N(\mu_2, \sigma_2) = N \left( \frac{k_1 L_1^2 + k_2 L_2^2}{L_1^2 + L_2^2}, \sqrt{\frac{1}{L_1^2 + L_2^2}} \right).
\]

(A-4)

So, eqn (A-2) becomes

\[
\int_{-\infty}^{\infty} k R_c(k) dk = A \sqrt{\frac{2\pi}{L_1^2 + L_2^2}} \frac{k_1 L_1^2 + k_2 L_2^2}{L_1^2 + L_2^2},
\]

(A-5)

or,

\[
\int_{-\infty}^{\infty} k R_c(k) dk = A \sqrt{2\pi} \frac{k_1 L_1^2 + k_2 L_2^2}{(L_1^2 + L_2^2)^{3/2}}.
\]

(A-6)

For the denominator of eqn (8) we have:

\[
\int_{-\infty}^{\infty} R_c(k) dk = A \int_{-\infty}^{\infty} \exp \left( -\frac{1}{2} (L_1^2 + L_2^2) \left[ (k - \frac{k_1 L_1^2 + k_2 L_2^2}{L_1^2 + L_2^2})^2 \right] \right) dk.
\]

(A-7)

Equation (A-7) is of the form

\[
\int_{-\infty}^{\infty} \exp(-ax^2) dx = \sqrt{\pi/a},
\]

and reduces to:

\[
\int_{-\infty}^{\infty} R_c(k) dk = A \sqrt{\frac{2\pi}{L_1^2 + L_2^2}}.
\]

(A-8)

Substituting eqn (A-8) and eqn (A-6) into eqn (11), we obtain the expression for the centroid or center frequency of the precompression power spectrum that is given below:

\[
f_{c1} = \frac{c}{2} \frac{k_1 L_1^2 + k_2 L_2^2}{\sqrt{L_1^2 + L_2^2}}.
\]

(A-9)

**APPENDIX B**

*Effect and use of bandwidth variation for strain estimation*

As is the case in velocity estimation, compression of the signal leads to two major changes in the postcompression power spectrum. Firstly, it causes the centroid shift, which in this paper we use as a direct strain estimator and, secondly, it results in a bandwidth variation. In this appendix, we examine the effect of this bandwidth variation to the centroid strain estimation and show how the estimation of this effect constitutes a direct estimator of strain.

Figure B1 considers the simple example of a rectangular spectrum. The centroid before compression is found at:

\[
f_{c1} = f_1 + \frac{B_1}{2},
\]

(B-1)

where \(c_1\) denotes the centroid, \(f_1\) is an extreme frequency and \(B_1\) is the bandwidth of the precompressed spectrum. After compression, the centroid is found at:

\[
f_{c2} = f_2 + \frac{B_2}{2},
\]

(B-2)

where \(c_2\) denotes the centroid, \(f_2\) is an extreme frequency and \(B_2\) is the bandwidth of the postcompressed spectrum.

The centroid shift is then found to be:
\[ f_{r2} - f_{r1} = f_2 - f_1 + \frac{1}{2}(B_2 - B_1). \]  
(B-3)

To estimate the bandwidths in eqn (B-3), the standard deviation of the precompression signal, eqn (A-4), is given by:

\[ \sigma_i = \frac{1}{\sqrt{L_i^2 + L_e^2}}, \]  
(B-4)

and the respective bandwidth of the precompression signal is equal to:

\[ B_i = \frac{c}{2\pi} \frac{2.35}{\sqrt{L_i^2 + L_e^2}}. \]  
(B-5)

Similarly, we find that the bandwidth of the postcompression signal is given by:

\[ B_i = \frac{c}{2\pi} \frac{2.35}{\sqrt{L_i^2 + \alpha^2 L_a^2}}, \]  
(B-6)

or

\[ B_i = \frac{c}{2\pi} \frac{2.35\alpha}{\sqrt{\alpha^2 L_a^2 + L_e^2}}. \]  
(B-7)

By considering eqn (B-3), (B-5) (B-6) and (B-7), we can conclude that compression does not result to a pure frequency shift. Furthermore, the bias, which is equal to the bandwidth change, depends on the strain incurred. When the strain is zero (i.e., \(\alpha = 1\)), from eqns (B-5) (B-6) and (B-7), we have:

\[ B_1 = B_2 \]  
(B-8)

and, therefore,

\[ f_{r2} - f_{r1} = f_2 - f_1. \]  
(B-9)

However, eqn 11 also shows that, when the strain is zero, no centroid shift occurs and, thus, both sides of eqn (B-9) nullify. When the strain is compressive (i.e., positive and \(\alpha > 1\)), spectral broadening occurs and the bias term is positive \((B_2 > B_1)\), but tension results in spectral bandwidth reduction and the bias term becomes negative \((B_2 < B_1)\). A more general expression of eqn (B-3) is as follows:

\[ f_{r2} - f_{r1} = \Delta f + \beta(s), \]  
(B-10)

where \(\Delta f\) denotes a pure frequency shift and

\[ \beta(s) = B_1 - B_1, \]

which is a function of strain \(s\) as eqns (B-5) (B-6) and (B-7) show.

Another interesting aspect of spectral broadening is that the bandwidth expansion can be used as a direct measure of strain because, from eqns (B-5) (B-6), (B-7) and \(\alpha^2 L_a^2 \sim L_a^2\), we have:

\[ \frac{B_2 - B_1}{B_1} = 2.35(\alpha - 1). \]  
(B-11)

From eqns (B-11) and the general form of (3), we have:

\[ s = \frac{B_2 - B_1}{1.35 B_1 + B_2}. \]  
(B-12)

Equation (B-11) with the small strain approximation of eqn (3) yields:

\[ s = 0.4 \frac{B_2 - B_1}{B_1}, \]  
(B-13)

which is a more straightforward estimator, very similar to the frequency shift estimator of eqn (18).