WAVELET DENOISING OF DISPLACEMENT ESTIMATES IN ELASTOGRAPHY

UDOMCHAI TECHAVIPOO* † and TOMY VARGHESE* ‡ *
Departments of *Medical Physics, † Electrical and Computer Engineering, and ‡ Biomedical Engineering,
The University of Wisconsin-Madison, Madison, WI, USA
(Received 12 September 2003; revised 18 November 2003; in final form 23 November 2003)

Abstract—Wavelet shrinkage denoising of the displacement estimates to reduce noise artefacts, especially at high overlaps in elastography, is presented in this paper. Correlated errors in the displacement estimates increase dramatically with an increase in the overlap between the data segments. These increased correlated errors (due to the increased correlation or similarity between consecutive displacement estimates) generate the so-called “worm” artefact in elastography. However, increases in overlap on the order of 90% or higher are essential to improve axial resolution in elastography. The use of wavelet denoising significantly reduces errors in the displacement estimates, thereby reducing the worm artefacts, without compromising on edge (high-frequency or detail) information in the elastogram. Wavelet denoising is a term used to characterize noise rejection by thresholding the wavelet coefficients. Worm artefacts can also be reduced using a low-pass filter; however, low-pass filtering of the displacement estimates does not preserve local information such as abrupt change in slopes, causing the smoothing of edges in the elastograms. Simulation results using the analytic 2-D model of a single inclusion phantom illustrate that wavelet denoising produces elastograms with the closest correspondence to the ideal mechanical strain image. Wavelet denoising applied to experimental data obtained from an in vitro thermal lesion phantom generated using radiofrequency (RF) ablation also illustrates the improvement in the elastogram noise characteristics. (E-mail: tvarghese@wisc.edu) © 2004 World Federation for Ultrasound in Medicine & Biology.

Key Words: Elastography, Elastogram, Elasticity, Imaging, Modulus, Strain, Ultrasound, Wavelet shrinkage, Denoising, Discrete wavelet transform, Radiofrequency ablation.

INTRODUCTION

Pathologic changes in tissue are generally related to the changes in tissue stiffness (Fung 1981). For example, scirrhous carcinoma of the breast appears as hard nodules, and liver tissue with cirrhosis is known to be stiffer than healthy liver tissue (Anderson 1953). However, these pathologic changes in tissue may not significantly alter tissue echogenic properties; thus, making it difficult to detect these conditions with conventional ultrasound (US). However, changes in tissue stiffness can be detected by imaging techniques based on estimating elastic attributes of tissue (Krouskop et al. 1987; Bertrand et al. 1989; Lerner et al. 1990; O’Donnell et al. 1991; Ophir et al. 1991; Cespedes 1993; O’Donnell et al. 1994; Bamber and Bush 1996; Varghese and Ophir 1996; de Korte et al. 1997; Garra et al. 1997; Ryan and Foster 1997; van der Steen et al. 1998; Varghese et al. 1998; Catheline et al. 1999; Lorenz et al. 1999; Pesavento et al. 1999; Insana et al. 2000; Ophir et al. 2000; Hiltawsky et al. 2001; Souchon et al. 2002; Varghese et al. 2002). One of these techniques, referred to as elastography (Ophir et al. 1991; Cespedes 1993; Ophir et al. 2000; Varghese et al. 2002), estimates the local strain or stiffness variations corresponding to small externally applied quasi-static compressions. Local tissue displacements are estimated using a time-domain cross-correlation between gated pre- and postcompression radiofrequency (RF) echo signals. The gradient of the tissue displacements in the axial direction provides an estimate of local tissue strains. These local strains are displayed as grey-scale images, termed elastograms.

Resolution is an important parameter of imaging modalities. In elastography, the cross-correlation window length and the amount of overlap in the data segments between successive windows are two signal-processing parameters that closely impact axial resolution.
(Cespedes 1993; Bilgen and Insana 1997; Alam et al. 2000; Righetti et al. 2002). The length of the cross-correlation window at a fixed overlap was initially used as a measure of the axial resolution in elastography (Cespedes 1993). Alam et al. (2000) demonstrated that axial resolution can be expressed as a bilinear function of the window length and the overlap, with the overlap being the more prominent factor. Long-duration windows are preferred for improved sensitivity, dynamic range and elastographic signal-to-noise ratio (SNRe); however, the elastogram is degraded due to poor resolution. Increasing the window length leads to a loss of resolution in the image. Increasing the overlap, on the other hand, improves the resolution of the elastogram, with the disadvantage of reduced SNRe due to increased noise contributions. Previous results indicate that a 50% overlap in the data segments for applied strains less than 5% provides the best compromise for elastography (Bilgen and Insana 1997; Varghese et al. 1998). The best noise performance is obtained with no overlapping of the data segments, which is not practical for imaging. Recently, Righetti et al. (2002) demonstrated that axial resolution in elastography is ultimately limited by and directly proportional to the wavelength (i.e., inversely proportional to the fractional bandwidth) of the US system.

An increase in the axial resolution without changing system parameters and the window length is, therefore, possible only by increasing the overlap beyond 50%. This involves a trade-off between SNRe and resolution in the elastogram (Varghese et al. 2001). Large signal overlaps introduce correlated noise patterns in the displacement estimates, leading to the so-called “worm” artefact observed in elastograms (Cespedes 1993; Ophir et al. 1999). This artefact appears as short and thin horizontal structures that do not propagate along the entire width of the elastogram (Cespedes 1993; Ophir et al. 1999). In this paper, we propose a technique using wavelet-shrinkage denoising, for reduction of these noise patterns from the displacement estimates before strain estimation. The technique described in this paper allows generation of quality elastograms with high axial resolution without a large drop-off in the SNRe.

Unlike Fourier basis functions that characterize the entire time interval, wavelets employ basis functions whose support is contained within any interval length, no matter its duration (Mallat 1998). Therefore, compactly supported wavelet basis functions can model local signal behavior efficiently because they are not constrained by properties of the signal far away from the location of interest. This property makes wavelet analysis suitable for signals that have abrupt transitions or localized phenomena (Resnikoff and Wells 1998). Wavelet denoising (Stein 1981; Vidakovic 1999; Jansen 2001) is a term used to characterize noise rejection by setting thresholds for wavelet coefficients, which form the contribution of each wavelet basis to the signal. Wavelet coefficients smaller than the threshold are set to zero and other coefficients are shrunk by the threshold or left untouched, depending on the algorithm used. Donoho and Johnstone (1994, 1995) and Donoho (1995) present a wavelet shrinkage denoising algorithm used to suppress Gaussian noise. They propose methods that select wavelet coefficient thresholds depending on the variance of the noise components in the signal. The efficiency of this denoising algorithm relies on the choice of wavelet basis, estimation of noise level, threshold selection method and parameters specific to the application. In elastography, the interface or boundary between two materials with different stiffness will appear in the displacement estimates as an abrupt change in the slope. Wavelet denoising reduces possible noise artefacts without smoothing the interface or boundary between the two regions.

In this paper, we propose the use of a wavelet-shrinkage denoising technique to reduce noise in displacement estimates obtained at high overlaps (around 90%). This technique improves the elastographic SNRe (Cespedes 1993; Varghese and Ophir 1997b; Varghese et al. 1998) and contrast-to-noise ratio (CNRc) (Varghese and Ophir 1998; Bilgen 1999) while maintaining the high axial resolution (Alam et al. 2000; Righetti et al. 2002) obtained due to the ability to use larger overlaps (Alam et al. 2000) in the data segments. The performance of the denoising technique is evaluated using data from simulations obtained using a 2-D analytic phantom (Ponnekanti et al. 1995; Kallel et al. 1996) and experimental data obtained from a thermal-lesion phantom generated using RF ablation.

**METHODS**

We present two techniques for wavelet-shrinkage denoising (Stein 1981; Vidakovic 1999; Jansen 2001) along with elastograms obtained using a low-pass filter on the displacement estimates. The unbiased risk estimator (SURE) of Stein (1981) and the Universal (Jansen 2001) threshold selection algorithms will be used for wavelet denoising. These threshold selection algorithms are two of the more popular choices in the wavelet literature. They utilize shrinkage operators (soft thresholding) and minimize a risk function. Both quantitative and qualitative comparison of results of these algorithms will be evaluated. A detailed discussion of wavelet transforms and wavelet filter banks are included in Appendix 1, and the threshold selection methods are presented in Appendix 2.

The algorithm for wavelet denoising can be broken down into three discrete steps. In the first step, the SDs
of the noise, (i.e., false peak or peak hopping errors and jitter noise) in the displacement estimates are estimated. A distribution of the false peak errors in the displacement estimates using a simulated uniformly elastic phantom is illustrated in Fig. 1. Note that these displacement errors can be approximated by a normal curve, thereby satisfying the assumption of a normal distribution of the noise used in the computation of the threshold in the SURE technique. The threshold selection criteria are discussed in more detail in Appendix 2. In the second step, each column of displacement estimates are decomposed independently using discrete wavelet transformation up to three levels. Finally, wavelet shrinkage is used to denoise each of the three levels of wavelet coefficients independently using the Universal and SURE thresholds. The length of the displacement estimates obtained with 90% overlap dictated the choice of three denoising levels. The displacement vector length was about 128 points and, therefore, the third level of wavelet coefficients was only 16 points, which was about the same size as the filters used in this paper. In general, more levels of denoising may be used depending on the length of the displacement vector. Displacement estimates are then reconstructed from denoised displacement coefficients.

The wavelet filter bank used in this paper is a symlets wavelet (sym8, following the MATLAB definition) with 16-point filter length (some papers refer to this filter as sym16). This wavelet filter bank was selected based on performance tests using the simulated single-inclusion analytic phantom. The filter banks were ranked by the maximum correlation coefficients of the strain estimates passing through the center of the inclusion along the axial and lateral directions between the elastogram, generated from the denoised displacement estimates and the ideal strain image. A total of 47 wavelet filter banks available on MATLAB version 6 with filter lengths less than or equal to 18 points were evaluated. These wavelet filter banks include Daubechies (db), Coiflets (coif), Symlets (sym), biorthogonals (bior), and reverse biorthogonals (rbio). The top 12 wavelet filters ranked on the basis of the correlation coefficients are shown in Table 1. Most of these filter banks yield elastograms that are a close match to the ideal mechanical strain image.

### Simulations

**Method.** Monte-Carlo simulation results using a single-inclusion analytic phantom were used to demonstrate

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![Figure 1. A histogram and its normal-distribution approximation](image)

**Table 1.** Top 12 wavelet filters ranked on the basis of the correlation coefficients in the axial and lateral directions.

<table>
<thead>
<tr>
<th>RANK</th>
<th>Filter name*</th>
<th>Correlation coefficient</th>
<th>Filter name</th>
<th>Correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>bior5.5</td>
<td>0.9919</td>
<td>rbio1.3</td>
<td>0.9987</td>
</tr>
<tr>
<td>2</td>
<td>db8</td>
<td>0.9913</td>
<td>rbio3.5</td>
<td>0.9981</td>
</tr>
<tr>
<td>3</td>
<td>rbio5.5</td>
<td>0.9912</td>
<td>db6</td>
<td>0.9979</td>
</tr>
<tr>
<td>4</td>
<td>coif2</td>
<td>0.9910</td>
<td>rbio1.5</td>
<td>0.9978</td>
</tr>
<tr>
<td>5</td>
<td>sym8</td>
<td>0.9908</td>
<td>db4</td>
<td>0.9978</td>
</tr>
<tr>
<td>6</td>
<td>rbio1.3</td>
<td>0.9906</td>
<td>db8</td>
<td>0.9976</td>
</tr>
<tr>
<td>7</td>
<td>bior4.4</td>
<td>0.9904</td>
<td>sym5</td>
<td>0.9975</td>
</tr>
<tr>
<td>8</td>
<td>sym6</td>
<td>0.9901</td>
<td>sym2</td>
<td>0.9968</td>
</tr>
<tr>
<td>9</td>
<td>rbio6.8</td>
<td>0.9899</td>
<td>coif2</td>
<td>0.9968</td>
</tr>
<tr>
<td>10</td>
<td>rbio1.5</td>
<td>0.9899</td>
<td>sym4</td>
<td>0.9966</td>
</tr>
<tr>
<td>11</td>
<td>bior6.8</td>
<td>0.9898</td>
<td>db3</td>
<td>0.9966</td>
</tr>
<tr>
<td>12</td>
<td>db6</td>
<td>0.9895</td>
<td>sym8</td>
<td>0.9965</td>
</tr>
</tbody>
</table>

*biorthogonal; db = Daubechies; rbio = reverse biorthogonal; coif = coiflets; sym = symlets.
the effectiveness of wavelet denoising in suppressing elastographic worm artefacts. The pre- and postcompression RF signal pairs were generated as follows. Tissue displacement information was obtained using the 2-D analytic model (Kallel et al. 1996) subjected to a uniaxial compression of 1% under plane-strain boundary conditions, where the displacements in the elevational plane is minimal. If the strain in one direction is assumed to be much less than the strain in the two other orthogonal directions, the smallest strain can be ignored, and the part is said to be under plane-strain conditions. An ideal mechanical strain image using the 2-D model is illustrated in Fig. 2a. The dimensions of the simulated phantom were 40 × 40 mm with an inclusion that was 12 mm in diameter and 3 times stiffer than the background. However, the strain contrast obtained with a modulus contrast of 3 was about 2.02 due to the contrast-transfer-function (conTE) parameter (Ponnekanti et al. 1995; Kallel et al. 1996). The displacement information was then incorporated into an acoustic Raleigh scattering model to simulate tissue-scattering profiles before and after compression. RF signal frames were then generated by convolving the scattering profiles with a simulated 2-D Gaussian-modulated point spread function (PSF). We used a transducer center frequency of 5 MHz with a 50% band width. The simulated RF signals were digitized using a 48-MHz sampling frequency. The speed of sound in tissue was assumed to be constant at 1540 m/s. Finally, scaled additive noise generated by convolving band-limited random noise and the transducer PSF was added to the RF signals to obtain the desired sonographic SNR. Note that the SNR incorporates both electronic and quantization noise (Varghese and Ophir 1997b; Varghese et al. 1998).

Local tissue displacement information was estimated from simulated and gated pre- and postcompression RF signals, from the location of the normalized cross-correlation function peak (Cespedes 1993). Subsample peak locations were calculated using parabolic interpolation. A window length of 3 mm with a 90% overlap was used to gate the signals for estimating a range of tissue displacements, unless otherwise stated. Higher overlaps of 95% or 99% could also be used; however, they may not be as efficient due to the increased similarity in the echo signals. The 90% overlap used in this paper was chosen arbitrarily; however, our selection takes less computation time (when compared with either a 95% or 99% overlap) and the number of pixels per unit length in the width and height of the elastograms was about the same with the 90% overlap. A 7-point 1-D median filter was then used to reduce false-peak errors (Weinstein and Weiss 1984; Walker and Trahey 1995). Elastograms obtained from the simulated analytic phantom using both a 90% and 50% overlap in the data segments without temporal stretching are illustrated in Fig. 2. Note the increased contribution of the worm artefact in the elastogram with the 90% overlap (Fig. 2b) as opposed to that with the 50% overlap (Fig. 2c). This result is expected because the 50% overlap significantly reduces errors in the displacement estimates (Bilgen and Insana 1997; Varghese et al. 1998). However, for the case with 50% overlap, the elastogram appears coarser because the reduced overlap decreases the number of displacement estimates in the axial direction. On the other hand, the definition of the inclusion is clearer in the elastograms obtained using the 90% overlap. The objective of this paper was to maintain the excellent edge definition obtained with the 90% overlap, in addition to minimizing the worm noise artefacts. The estimated elastograms shown in Fig. 2 were obtained using an SNR of 30 dB.

In this paper, we compare four methods for generating axial-strain elastograms from the median-filtered displacement estimates. All the methods use a 3-point least square strain estimator (LSQSE) (Kallel and Ophir 1997) (the only exception being the results presented in Fig. 6) to finally generate the elastograms from the displacement estimates. Three of the methods differ only in the type of filter used to denoise the displacement estimates. The elastogram generated without the use of any of the filtering techniques is referred to as the baseline method or BL. In the second method, displacement estimates were denoised using a raised cosine linear-phase finite impulse response low-pass filter (Proakis 1995). This method is referred to the low-pass method or low pass filtering method. The third and fourth methods for denoising elastograms in this paper use wavelet shrinkage denoising to reject noise in the displacement estimates using two different threshold-selection criteria. These criteria are the Universal and SURE.

Quantitative improvements in the elastograms obtained using the different denoising methods described above are compared using statistical mean and SD values.
of the elastogram quality parameters such as SNRe and CNR_e over 100 independent realizations. The definitions of these parameters are as follows:

\[
\text{SNR}_e = \frac{\mu_e}{\sigma_e},
\]

(1)

where \(\mu_e\) and \(\sigma_e\) are mean and SD of the strains from the elastogram, respectively. This parameter is usually calculated from elastograms of a uniform elastic phantom to avoid strain variations caused by inhomogeneities embedded within the phantom. The definition for the CNR_e is given by:

\[
\text{CNR}_e (\text{dB}) = 20 \log_{10} \left( \frac{2(\mu_{e1} - \mu_{e2})^2}{(\sigma_{e1} - \sigma_{e2})^2} \right),
\]

(2)

where the subscripts 1 and 2 denote values from regions 1 and 2, respectively (Varghese and Ophir 1998). Regions 1 and 2 usually correspond to inclusion and the background region, respectively. Comparisons were also performed using the maximum value of the correlation coefficient along the central scan lines in the axial and lateral directions across the inclusion between estimated elastograms and ideal mechanical strain images. Note that small reductions in the correlation coefficient value (1 = perfect correlation or match) denote large changes in the noise properties of the elastograms.

RESULTS

A comparison of the denoising performance using different algorithms is illustrated in Fig. 3a for displacement estimates with no added noise along the center of the inclusion in the axial direction. Note that most of the techniques follow the displacement profile accurately; however, low-frequency ringing is apparent in the methods that employ the low-pass filter. This ringing is primarily due to the large and abrupt initial drop or transition in the displacement estimates. At low frequencies, the passband is flat, and the transition band rolls off in a cosine shape toward the stop band. The low-pass filter with the cut-off frequency that closely followed the displacement estimate (LP2) was used to obtain quantitative measurements in this paper. This low-pass filter cut-off frequency was selected so that the smoothing of the displacement estimates did not significantly diminish the edge information in the elastogram. The cut-off frequency and transition band of the low-pass filter used in this paper was 0.15 \(\pi\) radians/sample. The other low-pass filter (LP1) shown in Fig. 3a had a lower cut-off frequency. In general, the smoothness in the elastogram increases with a lowering of the cut-off frequency (with...
subsequent reduction in the higher frequency or detail information); however, the ringing artefact will also increase.

The power spectral density (PSD) plots illustrating the spectral content of the displacement estimates after application of the denoising algorithms are illustrated in Fig. 3b. Observe the high-frequency spectral information that was rejected from the displacement estimates (high-frequency signals > 0.15 \( \pi \) radian/sample) using the low-pass filter (Fig. 3b). Note that the high-frequency information is retained in the denoising algorithms that use the wavelet algorithm. The PSD estimate obtained was the average of 100 independent Fourier spectra of the displacement profiles shown in Fig. 3a.

The variation in the elastographic \( \text{SNR}_e \) vs. applied strain is illustrated in Fig. 4 for a uniformly elastic phantom without temporal stretching. As expected, the baseline (BL) method provided the worst performance in this group of methods evaluated due to presence of the worm artefacts. Note that wavelet denoising using the Universal threshold (WU) provides the best strain estimation performance, and the low-pass filtering provides the second best performance. The strain estimation results shown in Fig. 4 were obtained from a region that did not contain ringing noise artefacts produced by the low-pass filter to provide a fair comparison. Incorporation of the ringing artefacts into the \( \text{SNR}_e \) reduces the low pass filtering method strain estimation performance to a level lower than that obtained using the BL method. The performance of the wavelet denoising algorithm using SURE (WS), however, shows an interesting trend, with the strain-estimation performance worse than that with the low-pass filter for applied strains less than 2%. However, for strains larger than 2% the performance of the SURE (WS) improves and is similar to that produced by the Universal (WU) method.

This unexpected behavior in the SURE method (WS) is due to the threshold selection criteria used. The threshold selected by SURE depends on the noise in the local signal as opposed to the noise variance estimate obtained from the entire data used by the Universal method (Jansen 2001). Generally, SURE selects lower thresholds than the Universal method. The variance of the displacement estimates increases with applied strain and, at large strains, it is difficult to separate noise and signal in the wavelet domain. According to eqn (A10) and (A11) in Appendix 2, if the wavelet coefficients can be divided into two groups containing larger and smaller valued coefficients, respectively, setting the threshold equal to the largest coefficient of the smaller-valued group will minimize eqn (A10). However, if all of the wavelet coefficients are generally large with similar magnitudes, the threshold that minimizes eqn (A10) will be large to eliminate most of these coefficients for the smoothness criteria. This is the reason that SURE provides similar results as the Universal method at large strains (due to the increased variance in the displacement estimates).

Figure 5 presents the variation in the \( \text{CNR}_e \) obtained without temporal stretching as a function of the contrast between the inclusion and background for both hard and soft inclusions. SURE again provides the highest \( \text{CNR}_e \) when compared to the other methods. The low pass filtering method provides a slightly higher \( \text{CNR}_e \) than...
SURE for soft inclusions, with the SURE performing better for higher contrast hard inclusions. The BL method provides the lowest CNRe values of all the methods. The point to be noted from Figs. 4 and 5 is the significant improvement (a factor of 4) in the SNRe and CNRe obtained with wavelet denoising using the Universal threshold over that with the BL method. These improvements in the elastographic quality parameters are primarily due to the reduction in the worm-noise artefacts obtained at the 90% overlap in the data segments used to generate the elastograms.

Elastograms obtained from the simulated single-inclusion phantom model are illustrated in Figs. 6 to 8 for an SNR of 10 dB. Figure 6 was obtained using a gradient operation (2-point LSQSE) between the displacement estimates, and Fig. 7 presents results obtained using a 3-point LSQSE. Note the improvement in the noise and elastogram quality parameters with the 3-point LSQSE. Note the presence of worm artefacts particularly in the baseline elastograms in Figs. 6a and 7a. We observe significant reductions in the worm artefact with wavelet denoising using the Universal (WU) and SURE (WS) thresholds in Figs. 6c and d (and also in Fig. 7c and d), respectively. The reduction in the worm-noise artefacts is approximately on the same level as that obtained using a low-pass filter (low pass filtering method) in Figs. 6 and 7b. However, low pass filtering method introduces another artefact caused by ringing in the displacement estimates, at the top of the elastograms. The ringing occurs due to the suppression of the high-frequency components of the discontinuity at the top edge of the displacement estimates. The contrast and CNRe in the elastogram are shown at the bottom of the elastograms. Note that CNRe obtained using the Universal method are significantly higher than those from SURE and low pass filtering method. Median filtering of the displacement estimates reduces some of the random spikes caused by the false peaks (Weinstein and Weiss 1984; Walker and Trahey 1995) in the displacement estimates. The impact of false-peak artefacts is clearly observed in Fig. 8, where the median filtering stage was not used before strain estimation. The lower SNR of 10 dB was used in Fig. 8 because the false peaks are not as prevalent at higher SNR values.

Elastograms obtained using SNR of 30 dB are illustrated in Fig. 9. Elastograms in Figs. 6 to 8 contain more noise artefacts than those in Fig. 9 due to the lower SNR in the RF data. The strain profiles along the centers of the inclusion in axial and lateral directions from the estimated elastograms in Fig. 9 are presented in Fig. 10a and b along with the ideal strain profiles obtained from Fig. 2a. Note from Fig. 10 that the strain profiles obtained using wavelet denoising Universal and SURE methods provides the closest correspondence to the ideal strain profile. The low-pass filtering method and both wavelet denoising methods provide the similar definition of the inclusion edges; however, the ripple inside the...
inclusion using low pass filtering method is larger, which is clearly observed in Fig. 10a. Further lowering of the cut-off frequency of the low-pass filter, however, at the expense of edge definition in the elastogram can reduce this ripple.

The fidelity of the estimated strain profiles to the ideal strain profiles is obtained by computing the correlation coefficient of the estimated strain with the ideal strain. The quantitative comparison of the strain estimation performance of these methods is presented in Fig. 11, where the maximum values of the correlation coefficient between the ideal and the estimated strain profiles were plotted vs. the SNR, from 10 dB to the case with infinite SNR. As shown in the figures, the correlation coefficient obtained using the Universal method is significantly higher when compared to the other methods. Universal and SURE methods provide roughly the same correlation coefficient in the lateral direction (Fig. 11b) as low pass filtering method; however, they did much better in the axial direction (Fig. 11a). The poor performance of the low pass filtering method in the axial direction is due to presence of the ringing artefacts. Both wavelet methods are superior to the BL method in both the axial and lateral directions. In addition, SDs were significantly smaller, as shown by the error bars for SNR, ≥ 20 dB.

The performance of the wavelet denoising methods for varying inclusion diameters (using 90% overlap, 30-dB SNR, and lesion contrast of 3) is illustrated in Fig. 12. Note that the Universal method consistently provided better elastograms for the entire range of inclusion diameters, compared to the other methods. Figure 13 presents a quantitative comparison for different lesion contrasts (using 90% overlap, 30-dB SNR, and 12-mm inclusion diameter). Irrespective of the inclusion contrast, the Universal method provides superior and less noisy strain estimates than with the other methods.

Figure 14 presents elastograms obtained with temporal stretching (Cespedes 1993; Alam and Ophir 1997; Varghese and Ophir 1997a) along with a 90% overlap in the data segments. Temporal stretching improves the noise properties of the elastograms; however, the worm artefacts due to the increased overlap are still observed in the BL elastogram. For low pass filtering method, the artefact caused by ringing in the low-pass-filtered displacement estimates now appears at both ends of the elastogram. This is due to discontinuities on both edges of the displacement estimates introduced with temporal
Wavelet denoising in conjunction with temporal stretching provides the best elastograms, as illustrated in Fig. 14c and d. This is primarily due to the improvement in the noise properties obtained with temporal stretching, along with the rejection of worm noise artefacts. However, we have to mention that temporal stretching functions best under 1-D tissue motion/compression (Alam and Ophir 1997; Varghese and Ophir 1997a; Alam et al. 1998) conditions and for the simple geometry that is used in this paper. The performance of temporal stretching deteriorates rapidly under 3-D tissue motions and complex object shapes under practical elastographic imaging conditions.

Figures 15 and 16 summarize the results presented in this paper. The bar plots provide a comparison of the four methods using gradient or 2-point LSQSE (2 p) and 3-point LSQSE (3 p) with and without the 1-D 7-point median filtering of the displacement estimates for 1% applied compression. Figure 15 presents SNR_e estimates obtained using simulated uniformly elastic phantoms at 10 dB and 30 dB SNR_s. The use of the 3-point LSQSE provides a small improvement in the SNR_e of the strain estimates when compared to the gradient estimation. The impact of false peaks on strain estimation is clearly observed in Fig. 15a with an SNR_s of 10 dB. Note the significant improvement in the results obtained when the median filter is used to reduce the false peaks for all the methods. However, when the SNR_s is increased to 30 dB in Fig. 15b, median filtering does not provide any significant improvements. Wavelet denoising using the Universal threshold (WU) provides the best performance in both SNR_s situations. The low-pass filtering technique provides the second best SNR_e values followed by wavelet denoising using SURE thresholds (WS) and the BL method.
The improvement obtained using wavelet denoising schemes becomes more apparent on the CNR_e plots using a single-inclusion phantom, as illustrated in Fig. 16. Note that the Universal method provides a significant improvement in the CNR_e estimates (the error bars do not overlap with any of the other methods), followed by the SURE, low pass filtering method and BL methods, respectively. Thus, in situations other than those with uniformly elastic phantoms, the wavelet denoising techniques provide superior performance.

PHANTOM EXPERIMENTS

Method

Experimental validation of wavelet denoising is presented in this section using a RF ablation (RITA® model 1500 RF generator and StarBurst™ XL electrosurgical probe, Mountain View, CA) thermal lesion phantom. The thermal lesion was generated in vitro in a freshly excised lobe of canine liver tissue acquired from an unrelated study. The RF ablation electrode is inserted into the liver lobe, and the prongs deployed to generate a 3 cm thermal lesion, taking care to keep the prongs within the liver parenchyma. RF ablation of the target tissue was conducted for a 10-min heating duration and target temperature of 80°C at a 150-W power level. The lobe of liver tissue containing the thermal lesion with approximate dimensions of 40 mm by 40 mm and 20 mm thickness was encased in a gelatin cube (15.5 g/100 mL of 200 Bloom calfskin gelatin with 1 g/L of 18 μm mean diameter 4000-e glass beads) with dimensions of 80 mm. Liver tissue was encased in the gelatin phantom so that it provides a regular surface for compression.

Elastographic imaging of the thermal lesion was performed using an Aloka SSD 2000 (Aloka Co., Ltd., Tokyo, Japan) real-time scanner using a 5-MHz linear-array transducer. The US RF signals were digitized using a 12-bit data acquisition board (Gage Inc., Montreal, Quebec, Canada) at a sampling rate of 50 MHz. The system incorporates a stepper motor-controlled compression system and a compression plate. A personal computer controls the operation of the entire system. Elastograms were generated after RF ablation using compressive increments of 0.5%. The image plane was perpendicular to the direction of the probe. Wavelet denoising on displacement estimates estimated from RF pre- and postcompression signals was performed offline. A window length of 3 mm with a 90% overlap between data segments was used.

Fig. 13. Quantitative comparison of the correlation coefficients vs. different modulus contrast among BL, low pass filtering method, Universal (WU) and SURE (WS) methods in (a) axial and (b) lateral directions.

Fig. 14. Elastograms at 30-dB SNR_e with temporal stretching created by (a) BL, (b) low pass filtering method, (c) Universal (WU) and (d) SURE (WS) methods.
Results

The B-mode grey-scale image along with elastograms generated are illustrated in Fig. 17. The elastogram obtained without any denoising (BL), as expected, contains the worm artefact (Fig. 17b). Note that the use of temporal stretching in Fig. 17c provides a small improvement in the noise characteristics of the elastogram; however, the trade-off is the significant loss in the edge definition of the thermal lesion along with artefacts due to overstretching (Varghese et al. 2001) inside the thermal lesion, and inability of temporal stretching to account for lateral and elevational decorrelation effects under practical imaging conditions. Observe that the edges at the boundary between the thermal lesion and tissue and at the boundary between tissue and gelatin of the SURE and Universal method elastograms (Fig. 17e and f) are more defined than those obtained using the low pass filtering method (Fig. 17d) technique. Note that, for the low pass filtering method elastogram, the artefact caused by ringing in displacement appears near the bottom edge instead of the top edge as in the simulated data, due to the different order of processing the pre- and postcompression RF signals. The displacement estimates in the experiment increase with depth from 0 to maximum instead of increasing from negative to zero as in Fig. 2a. The experimental results also illustrate that the wavelet denoising algorithms provide elastograms with significant reduction in the worm noise artefacts without diminishing the edge definition around the thermal lesion in the elastograms.

DISCUSSION AND CONCLUSIONS

Wavelet denoising is shown in this study to significantly improve the elastogram quality, especially at large overlaps. Previously, we were limited to the use of a 50% overlap in the gated RF data segments. However,
large overlaps improve the elastographic detail or axial resolution (Alam et al. 2000). Axial resolution in elastography has been previously expressed as a bilinear function with a larger weight on the window overlap parameter (Alam et al. 2000). However, because the use of increased overlaps results in increased worm artefacts, wavelet denoising becomes an important tool to generate high-resolution and high-quality elastograms. Temporal stretching of the postcompression RF signals does improve elastogram quality at higher overlaps; however, the worm artefacts are always present. In addition, as shown with the experimental data, temporal stretching does not work as well under practical imaging conditions. In addition, because wavelet denoising operates on the displacement data, noise amplifications due to the derivative (gradient) operation are also reduced. Wavelet shrinkage denoising provides a smoother displacement estimate (while preserving local edge information) before generating the strain information.

We have shown that wavelet shrinkage denoising of the displacement estimates considerably improves the elastographic quality parameters, such as the SNR, and CNR, and the correlation between the ideal and estimated strain profiles in the elastograms. For the case with the 90% overlap, wavelet denoising using the Universal threshold provides significant improvements in the elastogram by almost completely eliminating the worm artefacts and preserving the axial edge information of the inclusion. The SNR, and CNR, parameters also improve significantly due to the reduction in strain-estimation variance caused by the reduction of the worm artefacts.

The assumption used in wavelet denoising regarding the normal distribution of the noise in the displacement estimates is shown to hold in displacement maps obtained using an uniformly elastic phantom. Therefore, after wavelet decomposition, the noise in the wavelet coefficients is also Gaussian and suitable for wavelet shrinkage. Because replacing the smallest coefficients with zero reduces the variance of the data at the cost of an increasing bias (Vidakovic 1999), the wavelet coefficients after shrinkage are slightly more biased, but they generate smoother displacement estimates. The bias in the displacement estimates, however, may not be reflected in the strain estimates (because strain is the gradient of the displacement estimates). A comparison of the thresholds selected between the Universal and SURE techniques illustrates that, in general, the universal threshold is higher than SURE threshold, thereby providing smoother signals after shrinkage. Because the displacement usually processes a high degree of smoothness, the Universal threshold is more suitable for denoising the displacement estimates than SURE. On the other hand, the use of the low-pass filter for denoising reduces the detail in the elastogram by smoothing or removing the high-frequency content above the specified cut-off frequency. In addition, the use of the low-pass filter also introduces the low-frequency ringing artefact at discontinuities in the displacement profile. This artefact can be reduced by additional processing by padding data at the edge of the displacement field to preserve continuity.

Wavelet denoising in the displacement estimates works well for different inclusion sizes and contrasts. In the case of 90% overlap and SNR, ≥ 20dB, we show significant improvement in the elastograms. Wavelet denoising, however, does not provide significant improvements in the elastogram estimated using a 50% overlap, and in situations with extremely low SNR, . For the 50% overlap case, the errors in the displacement estimates are already minimized, and wavelet denoising would not provide any significant additional benefits. The improvement using wavelet denoising will, therefore, be more noticeable for overlaps larger than 50%. For lower SNR, the denoising method struggles with large jumps in the displacement estimates caused by false peaks. Because the wavelet-denoising method preserves the edge information, the jumps are restored instead of suppressed. The median filtering stage used before wavelet denoising significantly improves the denoising performance; however, it does not remove all the noise spikes at low SNR,. Increasing the length of the median filter may provide improved performance, the trade-off being the loss of detail in the elastogram. The experimental results presented in this paper also illustrate the effectiveness of wavelet denoising in reducing the worm artefact.

Wavelet denoising provides excellent detectability and edge discrimination and the highest SNR, and CNR, values when compared to the other methods illustrated in this paper. Wavelet denoising also does not introduce any artefacts into the elastogram. Potential benefits include the significant reduction in correlated noise artefacts without favoring a particular set of frequencies as with the low-pass filter, that potentially could exclude high-frequency content in the displacement estimates.

Acknowledgments—This work was supported in part by start-up grant funds from the Department of Medical Physics, Medical School and Graduate School to Dr. Varghese at the University of Wisconsin-Madison and Whitaker Foundation (grant RG-02-0457).

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APPENDIX 1

A specific example of discrete wavelet decomposition using Haar basis functions (Mallat 1998) and wavelet transform using filter banks is presented. The interested reader is referred to the writing of Vidakovic (1999) and Jansen (2001) for further details on the algorithm used in this paper.

DISCRETE WAVELET DECOMPOSITION AND RECONSTRUCTION

Any discrete signal can be represented as a linear combination of Haar basis functions (the Haar wavelets are used only as a representative case and for discussion purposes only; the symlets wavelet filter bank is used for wavelet denoising in this paper). Suppose \( x(i) \) is a discrete signal with length \( N \) (i.e., \( n = \{1, 2, 3, 4, 5, 0, 1, 0\} \)). Taking pairs of neighbors, such as \( x(1) \) and \( x(2), x(3) \) and \( x(4) \), and so on, we then compute the first level average coefficients \( a_i(\tau) = (x(2) + x(3))/2 \) and difference coefficients \( d_i(\tau) = (x(2) − x(3))/2 \) of the average and difference coefficients of the next level are recursively computed from the previous average coefficients, and so on. The levels are indicated by subscripts. It can be shown that \( a_0 \) can be recovered from \( a_1 \) and \( d_1 \) at level 2, and finally \( x \) from \( a_1 \) and \( d_1 \).
respectively. For example, \( x(2i - 1) = a_i(i) + d_i(i) \) and \( x(2i) = a_{i+1}(i) - d_{i+1}(i) \) for \( i = 1, \ldots, M \) can be computed. From the above example, only the coefficients \( a_i, d_i, d_0, \) and \( a_0 \) are necessary to obtain a perfect reconstruction of the original signal \( x \). On the other hand, \( x \) can also be decomposed into a linear combination of Haar basis functions with the coefficients specified by \( d_1, d_2, d_3 \) and \( a_i \). In general, it can be rewritten as:

\[
x = \sum_{i=0}^{N-1} \langle x, \xi_i \rangle \xi_i,
\]

where the coefficient \( \langle x, \xi_i \rangle \) is an inner product of the arguments and \( \xi_i \) is a Haar basis function of index \( k \). For example, the basis function that gives the coefficient \( d_{i+1}(i) \) is \( \{1/2, -1/2, 0, 0, 0, 0, 0, 0\} \) and \( \{0, 0, 1/2, -1/2, 0, 0, 0, 0\} \) gives the coefficient \( d_i(i) \). This is also called the Haar transform.

The average coefficient \( a_i \) is the coarse or low frequency component of \( a_{i-1} \) and \( d_i \) is detail or high frequency component of \( a_{i-1} \), for \( a_0 = x = 1, 2, 3 \) and \( 3 \). The length of \( a \) and \( d \) is half of the length of \( a_{i-1} \). The decomposition and reconstruction of a discrete signal can also be performed using wavelet filter banks (Vidakovic 1999; Jansen 2001), as discussed in the next subsection.

### WAVELET TRANSFORMS AND FILTER BANKS

Let \( \{h_0, h_1, \tilde{h}_0, \tilde{h}_1\} \) be a set of wavelet filters. The low-pass and high-pass wavelet filters are indicated by subscripts 0 and 1, respectively. The symbols \( h \) and \( \tilde{h} \) denote the decomposition and reconstruction filters, respectively. For a discrete signal \( x \), wavelet decomposition of \( x \) at level \( j \), \( x_j \) for \( 0 \leq j \leq M \), can be expressed as:

\[
x_j = \left( x_{j+1} * h_0 \right) \downarrow 2 \quad \text{and} \quad x_j = \left( x_{j+1} * \tilde{h}_0 \right) \downarrow 2,
\]

where \( x_{j+1} \) and \( x_j \) are the low-frequency (coarse) and high-frequency (detail) components, respectively, at level \( j \). * denotes a convolution operation, and \( \downarrow 2 \) denotes decimation (i.e., down sampling) by 2. The finest-level signal \( x_0 \) is equal to \( x \). The maximum number of levels \( M \) is \( \log_2 N \), where \( N \) is the number of points in \( x \). The reconstruction of the low- and high-frequency components at level \( j \) can be expressed as:

\[
x_j = \left( x_{j+1} * h_0 \right) \uparrow 2 + \left( x_{j+1} * \tilde{h}_0 \right) \uparrow 2,
\]

where \( \uparrow 2 \) is dilation (i.e., up sampling) by 2. Note that the level index also can be defined by \( \log_2 N \), where \( N \) is the number of points in the signal. Therefore, the index from finest to coarsest levels will run from \( M \) down to zero.

In the above example for the Haar wavelet filter bank, \( h_0 = \tilde{h}_0 = \{1/2, 1/2\} \) and \( h_1 = \tilde{h}_1 = \{1/2, -1/2\} \). There are many families of wavelets (e.g., Haar, Daubechies, symlets, coiflets, biorthogonal and reverse biorthogonal waves), each of which might contain more than one set of wavelet filters.

### APPENDIX 2

Fundamental concepts about the wavelet shrinkage denoising algorithm are presented. A brief description of the universal threshold (Donoho and Johnstone 1994) and threshold selection using the Stein (1981) unbiased risk estimator or SURE (Donoho and Johnstone 1995) is also presented in this section.

### WAVELET SHRINKAGE DENOISING

Wavelet shrinkage is a technique that is used to reduce noise in the wavelet coefficients by shrinking the coefficients toward zero, depending on a preselected threshold. The shrinkage operator can be written as:

\[
\eta(x, \lambda) = \begin{cases} 0 & \text{if } |x| \leq \lambda \\ \text{sign}(x) (|x| - \lambda) & \text{if } |x| > \lambda \end{cases}
\]

For example, let \( \{1.1, -1.1, 0.5, -0.3\} \) be a set of wavelet coefficients and 0.5 be the selected threshold. Then, the coefficients after shrinkage are \( \{0.6, -0.6, 0, 0\} \). Equation (A4) will be used to define the objective function in the following paragraph. This method is also called soft thresholding.

Assume that the objective is to estimate the mean vector \( \mu \) from a multivariate normal distribution observation \( x \). Because the orthogonal wavelet transforms this observation into the same stochastic structure, the wavelet coefficients themselves can be considered as the observation. The objective function is the mean square error between the mean vector \( \mu \) and the value after shrinkage \( \hat{\mu}(x, \lambda) \) which can be written as:

\[
R(\lambda) = ||\hat{\mu}(x, \lambda) - \mu||^2.
\]

Note that the expectation of this function \( E[R(\lambda)] \) is called the risk function. The threshold \( \lambda \) has to be selected to minimize this objective or risk function.

Fig. 18. (a) The simulated signal of noisy rectangular pulses and its denoised versions using low pass filtering method, Universal (WU) and SURE (WS) methods compared with the ideal signal without noise.
THRESHOLD SELECTION METHODS

There are many methods for selecting the threshold for wavelet shrinkage. Two of the popular choices are the Universal and SURE thresholds.

Universal threshold.

The Universal threshold is obtained from the asymptotic behavior of the minimum risk threshold function. Suppose \( \mu \) is a piecewise polynomial and \( \lambda^* \) minimizes the ER(\( \lambda \)), then, for the number of points in signal \( N \rightarrow \infty \),

\[
\lambda^* \sim \sqrt{2 \log N} \sigma
\]

(A6)

(Jansen 2001). The Universal threshold is defined by:

\[
\lambda_{\text{UNIV}} = \sqrt{2 \log N} \sigma.
\]

(A7)

This threshold is valid for all signals with length \( N \) and it also removes noise with high probability contributing to the visual quality of reconstructed signals (Vidakovic 1999).

SURE threshold.

For a multivariate normal distribution of \( d \) dimensions, \( x \sim N(\mu, I) \), Stein (1981) introduced a method to estimate \( \| \hat{\mu}(x) - \mu \|^2 \), where \( \hat{\mu}(x) \) is a particular fixed estimator of \( \mu \). Stein also illustrated that an unbiased estimator of the risk function could be obtained. Let \( \hat{\mu}(x) = x + g(x) \), where \( g(x) \) is a weakly differentiable function, and then the risk function is:

\[
E_x(\| \hat{\mu}(x) - \mu \|^2) = d + E_x(\| g(x) \|^2 + 2 \nabla \cdot g(x)),
\]

(A8)

where:

\[
\nabla \cdot g(x) = \sum_{i=1}^{d} \frac{\partial}{\partial x_i} g_i.
\]

(A9)

It is interesting that the estimator \( \hat{\mu}(x) \) can be nearly arbitrary; for instance, it can be biased and nonlinear. Donoho and Johnstone (1995) used the above result with the wavelet shrinkage method to obtain:

\[
\text{SURE}(x, \lambda) = d - 2 \sum_{i=1}^{d} I(\| x_i \| \leq \lambda) + \sum_{i=1}^{d} (\| x_i \| \wedge \lambda)^2
\]

(A10)

where \( I(\| x_i \| \leq \lambda) \) is equal to 1 if \( \| x_i \| \leq \lambda \) and zero otherwise, and \( (\| x_i \| \wedge \lambda) \) is the minimum value between the arguments. The second term denotes the number of coefficient magnitudes that are less than the threshold, and the third term describes the energy loss (magnitude squared) after shrinkage. The expectation with respect to \( \mu \) of this function is an unbiased estimator of risk. Selecting a threshold that minimizes this estimator of risk, we obtain:

\[
\lambda_{\text{SURE}} = \arg \min_{\lambda > 0} \text{SURE}(x, \lambda)
\]

(A11)

Note that this threshold selection is based on the normal distribution with unit variance; therefore, the noisy signals to be denoised using this method must have normal model and be normalized to unit variance before denoising. For the algorithm used in this paper, we shrink only the first three finest levels of wavelet coefficients because the number of points in the signal for denoising is limited.

To illustrate the performance of wavelet denoising in the suppression of noise without blurring sharp edges, a simulated synthetic data set containing two noisy rectangular pulses with different pulse durations and amplitudes are denoised using the wavelet shrinkage algorithm and the raised cosine FIR low-pass filter with length of 20 points, and cut-off frequency of 0.15π radians/sample. The additive noise is Gaussian with SD of 0.02. The original and denoised rectangular pulses are plotted in Fig. 18. Note that the low-pass filter generates smoother signals; however, the sharp edges in the signal are blurred, with the ringing artefact clearly seen at the base of the pulses. In addition, the amplitudes of the pulses are erroneous in that, for the wider pulse, it is higher and, for the thinner pulse, it is lower than the ideal signal. In contrast, observe that, with wavelet shrinkage denoising, both threshold selection methods preserve the sharp edges with smaller ringing at the base of the pulse, and provide signal amplitudes closer to that of the ideal signal.