WISCONSIN
UNIVERSITY OF WISCONSIN–MADISON

Course name: **Mathematical Methods in Medical Physics**
Course #: **Medical Physics 573 / Biomedical Engineering 573**
Instructor: **Diego Hernando, PhD (dhernando@wisc.edu)**
Assistant: **TBD**
Session: **Fall 2022**
Credits: **3**
Lectures: **MWF 11:00AM-11:50AM (tentative)**
Office hours: **TBD**
Location: **WIMR 1022**
Canvas URL: **TBD**

Instructional mode: **Face-to-face**
Course designations and attributes: **Graduate level**
Contact hours: **3 hours/week**

Course description: Mathematical fundamentals required for medical physics and biomedical applications, including signal analysis and mathematical optimization.

Learning objectives:

Upon completion of this course, students should be able to:
- Summarize the utility of signal analysis in one and several dimensions
- Identify and apply convolutions and Fourier Transforms in one and several dimensions
- Apply the properties of the Fourier Transform in medical physics and other biomedical settings
- Illustrate the limitations of the Fourier transform, and recall the advantages of alternative signal analysis tools (eg: wavelet transform) **[grad only]**
- Distinguish between types of optimization problems, including convex vs non-convex, and unconstrained vs constrained
- Recognize the relative performance of basic optimization algorithms
- Formulate image reconstruction as an optimization problem
- Formulate therapy planning as an optimization problem
- Implement practical optimization algorithms using computational methods

ABET Outcomes:

1. an ability to identify, formulate, and solve complex engineering problems by applying principles of engineering, science, and mathematics

2. an ability to apply engineering design to produce solutions that meet specified needs with consideration of public health, safety, and welfare, as well as global, cultural, social, environmental, and economic factors

6. an ability to develop and conduct appropriate experimentation, analyze and interpret data, and use engineering judgment to draw conclusions

7. an ability to acquire and apply new knowledge as needed, using appropriate learning strategies.

Brief list of topics to be covered:
- Signal analysis basics
- Fourier Transform: continuous- and discrete-space
- Properties of the Fourier transform
- Fourier Transform in multiple dimensions
- Sampling in one and multiple dimensions
- Short Time Fourier Transform
- Wavelets in one and multiple dimensions
- Motivation for studying optimization
- Optimization formulations
- Optimization algorithms
- Applications in imaging
- Applications in therapy planning

Prerequisites: (MATH 234 and 319), (MATH 234 and 320), or MATH 376 and (PHYSICS 202 or 208), or graduate/professional standing.

Homework: Homework sets will be due every 2-3 weeks, and will include both theoretical derivations and proofs, as well as computational exercises. Graduate students will have an additional homework assignment covering the conceptual and/or computational aspects of the "grad only" learning objective (i.e., alternative non-Fourier signal analysis tools).

Exams: one midterm and one final exam. Both exams are in-class, open-note.

Grading: The midterm exam will count for 25% of the final grade. The final exam grade will count for 25%. Homework sets will count for 45%. In-class participation will count for 5%.

Late homework policy: Each student will get one free 2-day late pass. In other words, you can hand in a single homework set up to two days after the deadline, with no grade penalty. For example, if the homework was due on Friday at 11am, you could hand it in on Sunday at 11am. Once you use this free pass on one homework set, the deadline will be strictly enforced, i.e.: any subsequent homework sets handed in after the deadline will not be graded.

Course credit information: This class meets for three 50-minute class period each week over the fall/spring semester and carries the expectation that students will work on course learning activities (reading, writing, problem sets, studying, etc) for about 2 hours out of classroom for every class period. This syllabus includes additional information about meeting times and expectations for student work.

Required texts: No texts are required for this course. Some relevant texts and resources include:
- **Medical Imaging Signals and Systems** by Jerry L. Prince, Jonathan Links, Pearson Education, 2nd Edition, 2014.
- **The Fourier Transform & Its Applications** by Ronald Bracewell, McGraw-Hill, 3rd editiojn, 1999.
- **Introduction to the Mathematics of Medical Imaging** by Charles L. Epstein, SIAM, 2nd edition, 2007.
- **Random Processes for Engineers** by Bruce Hajek, Cambridge University Press, 2015.
- **Nonlinear Programming** by D. P. Bertsekas, Athena Scientific, Belmont, MA, 3rd edition, 2016.

- **Convex optimization** by S. Boyd, L. Vandenberghe, Cambridge University Press, 2004.
- **Magnetic Resonance Imaging: A Signal Processing Perspective**, by Z-P. Liang and P. Lauterbur, IEEE Press, New York, NY, 1999.
- **Computational methods for inverse problems**, by C.R. Vogel, SIAM, Philadelphia, PA, 2002.
- **Khan's Treatment Planning in Radiation Oncology**, Editors: Faiz M. Khan, Paul W. Sperduto, John P. Gibbons, Lippincott Williams & Wilkins, Philadelphia, PA, 5th edition, 2021.
- Additionally, Prof. Barry van Veen's excellent video lectures are available on the **AllSignalProcessing YouTube channel**: https://www.youtube.com/channel/UCooRZ0pxedi179pBe9aXm5A

Related courses: Med Physics / BME 574 describes probability and stochastic concepts that are complementary to the mathematical foundation provided in this course.

Regular and substantive interaction: This course has been designed to meet the requirements for regular and substantive student-instructor interaction. Specific modes of interaction include the in-person lectures (three 50-minute sessions per week) and weekly office hours. In addition, the instructor will provide lecture notes (largely self-contained) as well as computational examples (in the form of Jupyter notebooks) to implement the concepts studied in class. These Jupyter notebooks are easily accessed by the students using a web browser through a JupyterHub. Finally, the homeworks will assess the students' understanding of the theoretical and computational components of the course.

Grading rubric:

90.0-100.0: A,

85.0-89.9: AB

80.0-84.9: B

75.0-79.9: BC

70.0-74.9: C

60.0-69.9: D

<60.0: F

Policy for excused/unexcused absence: Students are expected to attend all course lectures. However, reasonable accommodations will be extended to students who are unable to attend specific lectures. Permission for any absence should be requested from the instructor at least 3 days in advance, except in those cases when this advance request is inherently infeasible.

Academic Integrity:

By enrolling in this course, each student assumes the responsibilities of an active participant in UW-Madison's community of scholars in which everyone's academic work and behavior are held to the highest academic integrity standards. Academic misconduct compromises the integrity of the university. Cheating, fabrication, plagiarism, unauthorized collaboration, and helping others commit these acts are examples of academic misconduct, which can result in disciplinary action. This includes but is not limited to failure on the assignment/course, disciplinary probation, or suspension. Substantial or repeated cases of misconduct will be forwarded to the Office of Student Conduct & Community Standards for additional review. For more information, refer to studentconduct.wiscweb.wisc.edu/academic-integrity/.

Accommodations for Students with Disabilities:

McBurney Disability Resource Center syllabus statement: "The University of Wisconsin-Madison supports the right of all enrolled students to a full and equal educational opportunity. The Americans with Disabilities Act (ADA), Wisconsin State Statute (36.12), and UW-Madison policy (Faculty Document 1071) require that students with disabilities be reasonably accommodated in instruction and campus life. Reasonable accommodations for students with disabilities is a shared faculty and student responsibility. Students are expected to inform faculty [me] of their need for instructional accommodations by the end of the third week of the semester, or as soon as possible after a disability has been incurred or recognized. Faculty [I], will work either directly with the student [you] or in coordination with the McBurney Center to identify and provide reasonable instructional accommodations. Disability information, including instructional accommodations as part of a student's educational record, is confidential and protected under FERPA." http://mcburney.wisc.edu/facstaffother/faculty/syllabus.php

Diversity & Inclusion:

Institutional statement on diversity: "Diversity is a source of strength, creativity, and innovation for UW-Madison. We value the contributions of each person and respect the profound ways their identity, culture, background, experience, status, abilities, and opinion enrich the university community. We commit ourselves to the pursuit of excellence in teaching, research, outreach, and diversity as inextricably linked goals.

The University of Wisconsin-Madison fulfills its public mission by creating a welcoming and inclusive community for people from every background – people who as students, faculty, and staff serve Wisconsin and the world." https://diversity.wisc.edu/

Academic Calendar & Religious Observances:

See: https://secfac.wisc.edu/academic-calendar/#religious-observances

Course Evaluations:

Students will be provided with an opportunity to evaluate this course and your learning experience. Student participation is an integral component of this course, and your feedback is important to me. I strongly encourage you to participate in the course evaluation.

Digital Course Evaluation (AEFIS)

UW-Madison now uses an online course evaluation survey tool, AEFIS. In most instances, you will receive an official email two weeks prior to the end of the semester when your course evaluation is available. You will receive a link to log into the course evaluation with your NetID where you can complete the evaluation and submit it, anonymously. Your participation is an integral component of this course, and your feedback is important to me. I strongly encourage you to participate in the course evaluation.

Detailed Contents:

1. Signal Analysis: Fourier and Wavelet Theory and Applications	
Theory	***Computation***

1. Signal analysis basics (0.5 weeks)
2. Linear shift-invariant systems (0.5 weeks)
3. Continuous Fourier Transform (0.5 weeks)
4. Discrete Fourier Transform and Fast Fourier Transform (0.5 weeks)
5. Basic properties of the Fourier transform (1 week)
    a. Convolution,
    b. Central Slice Theorem,
    c. Symmetry, etc.
6. Fourier Transform in multiple dimensions (1 week)
7. Sampling in one and multiple dimensions (1 week)
8. Applications of the Fourier Transform (1 week)
    a. Convolution
    b. Image reconstruction
9. Short Time Fourier Transform (0.5 weeks)
10. Wavelets in one and multiple dimensions (1 week)
    a. Motivation
    b. Description
    c. Applications
	• FFT
	• Convolution
	• Convolution examples in therapy planning (dose calculation)
	• Multi-dimensional Fourier
	• Central Slice Theorem examples
	• Filtering
	• Magnitude vs phase information in Fourier space
	• Aliasing and artifacts when representing/reconstructing an image from limited Fourier samples
	• Image compression concepts
	• Wavelet transforms and applications

Midterm exam

2. Fundamentals of Optimization	
Theory	***Computation***

1. Motivation for studying optimization (1 week)
    a. Imaging
    b. Therapy
2. Formulations (2 weeks)
    a. Constrained vs unconstrained
    b. Linear vs non-linear
    c. Convex vs non-convex
    d. Continuous vs discrete
3. Algorithms (2 weeks)
    a. Closed form vs iterative
    b. Optimality conditions
    c. Various types of iterative algorithms
    d. Convergence: global vs local, speed of convergence
4. Applications (2 weeks)
    a. Image reconstruction and processing
    b. Therapy planning
	• Linear and non-linear least-squares
	• Total least squares
	• Optimization examples from
	    o Imaging
	    o Therapy
	• Regularized formulations
	    o L1
	    o L2
	• Compare algorithms
	    o Gradient Descent
	    o Newton's method
	    o Quasi-Newton methods
	    o Stochastic Gradient Descent

Final exam

Related courses at other institutions:

UCL: MPHY3893 - Mathematical Methods in Medical Physics (Jem Hebden)
http://www.ucl.ac.uk/medphys/prospective-students/modules/mphy3893

UT: GS02 1183 – Applied Mathematics in Medical Physics
http://www.uthgsbsmedphys.org/GS02-0183/Syllabus%20Math%20Med%20Phys%2009-07-2012.pdf

U Chicago: MPHY 34900 - Mathematics for Medical Physics
http://medicalphysics.uchicago.edu/program/descriptions.html

Duke: MP 530 - Modern Medical Diagnostic Imaging System.
https://medicalphysics.duke.edu/courses

Stanford: EE369C - Medical Image Reconstruction (Pauly)
http://web.stanford.edu/class/ee369c/index.html

USC: EE 592 - Computational Methods for Inverse Problems (Haldar)
https://web-app.usc.edu/soc/syllabus/20173/30794.pdf

Michigan: Bioengineering/Math 464 – Inverse Problems
http://bme.umich.edu/course/biomede-464/

Michigan: EECS 755 - Topics in Signal Processing: Model-based image reconstruction methods (Fessler)
https://web.eecs.umich.edu/~fessler/course/755/index.html