Computed Tomography Notes, Part 2

Practical Considerations in CT

Sampling

CT (the “C” meaning “computed”) relies on the sampling of projection data for processing on a computer. There are 2 kinds of sampling – radial sampling (along the projection in \( R \)) and angular sampling (\( \theta \)).

Sampling in \( R \) (e.g. \( \Delta x \)) depends on the spatial frequency content that you wish to represent in your final image. Subsampling in this domain results in the relatively benign spectral aliasing. Sampling in \( \theta \) is a more complex, though, since it is really a sampling in the Fourier domain of the object and as such subsampling will result in aliasing of spatial information in the image.

Recall, that the minimum sampling distance in the Fourier domain is dictated by the field of view. For example,

\[
\Delta k_{\text{max}} \leq \frac{1}{FOV} = \frac{1}{2R_0}
\]

where \( R_0 \) is the maximum extent of the object. Also, recognize that

\[
\Delta k_{\text{max}} = \Delta \theta \cdot \rho_{\text{max}}
\]

but the maximum spatial frequency that can be represented by a discretely sampled projection is:
\[ \max = \frac{1}{2\Delta x} \]

and thus:

\[ \Delta \theta \leq \frac{\Delta x}{R_0} \]

For example, if our device has 512 samples across the field of view \( \frac{2R_0}{\Delta x} = 512 \), then:

\[ \Delta \theta \leq \frac{1}{256} \]

and the number of projections will be:

\[ N_{proj} = \frac{\theta_{max} - \theta_{min}}{\Delta \theta} = 256\pi = 804 \]

Thus approximately 804 projections would be required to fully sample the object for a final image of size roughly 512 by 512. Of course, we can always sample fewer angles and filter (smooth) the data to reduce \( \rho_{\text{max}} \) which reduces spatial resolution.

**Fan-Beam Geometry**

Our analysis of CT to date was for a parallel ray x-ray source, while in practice, we have a small source that projects a fan of x-rays:

For a given angle of the CT gantry, we actually collect a projection where the angle of projection varies as a function of \( R \). Observe that if the fan width is \( \phi \) then the maximal \( R \)
location will actually have an angle of \((\theta + \phi/2)\), while the \(R = 0\), will have an angle of \(\theta\) and the minimal \(R\) will have an angle of \((\theta - \phi/2)\):

Question: Does the 1D FT of this projection result in a curved line in the Fourier domain? No – it results in a curved line in the sinogram space (Radon space):

In order to be able to reconstruct the image we need to fill in the Radon space completely (recall the FT of horizontal lines will give lines in the Fourier domain and incomplete lines will limit
our ability to fill in the Fourier domain). In order to fully sample the Radon space, we will need to sample projections over angles $\theta \in [-\varphi / 2, \pi + \varphi / 2]$ or over a $(\pi + \varphi)$ range.

One way to reconstruct this data will be to resample the above to:

and then reconstruct using the usual methods. There are also filtered backprojection methods and iterative (e.g.) ART formulations that deal directly with fan beam data (see Kak book, for example).

**Circular Convolution**

When performing filtering, it is often convenient to do this filtering in the Fourier domain.
One problem that arises is circular convolution. Because the DFT assumes that the object is periodic, the convolution function (which has long tails) will extend into the replicated versions of the object:

The typical solution to this problem is to zero pad the object prior to Fourier transformation for filtering:

**Beam Hardening**

Beam hardening manifests itself as an apparent decrease in $\mu$ as the beam passes through the object. Thus, for short paths through the object (e.g., near the edge) the apparent $\mu$ is higher than for long paths through the object (e.g., near the center):
One effect of this is a “cupping” of the images of $\mu$, which results in an apparent reduction in image intensity at the center of the object: