This is quiz 0. It will not be graded. It is intended to help me identify the range of expertise reflected in the class. Please provide short concise answers. Forgo rigorous proof. "I don’t know" is okay if you really have no idea, but please don’t do this unless you really don’t have the slightest idea how to start.

Figure 1:

1. Consider Figure 1 above. Assume the phase is in units of radians, and the amplitudes of the real part and magnitude are unitless. What is the mathematical form of the complex function? Sketch the approximate value of the function in polar coordinates for the position indicated by the arrow (→), near x = 0.01 cm.

Observations:

(i) Magnitude is unity

(ii) Real part is \( \cos \left( \frac{\pi x}{0.02} \right) = \cos \left( \frac{\pi x}{0.01} \right) \)

(iii) \( \Phi(x) = 2\pi \frac{x}{0.02} = \frac{\pi x}{0.01} \)

Short Answer => \( f(x) = e^{-i \Phi(x)} = e^{-i \frac{\pi x}{0.01}} \)  
\( f(0.01 \text{ cm}) = e^{-i \pi} = -1 \)

Longer Answer => \( \Phi(x) = \tan^{-1} \left[ \frac{\text{Im}}{\text{Re}} \right] \) or \( \frac{\pi x}{0.01} = \tan^{-1} \left[ \frac{\text{Im}}{\cos \left( \frac{\pi x}{0.01} \right)} \right] \)

\( \text{Im} = \tan \left( \frac{\pi x}{0.01} \right) \cos \left( \frac{\pi x}{0.01} \right) = \frac{\sin \left( \frac{\pi x}{0.01} \right)}{\cos \left( \frac{\pi x}{0.01} \right)} \)

\( = \sin \left( \frac{\pi x}{0.01} \right) \), and finally: \( f(x) = \cos \left( \frac{\pi x}{0.01} \right) + i \sin \left( \frac{\pi x}{0.01} \right) \)

\( f(0.01 \text{ cm}) = \cos (\pi) + i \sin (\pi) = -1 \)
2. Consider Figure 2a and b above. Assuming 2b, the lower panel, is the real part of the continuous Fourier transform of the rect function in 2a, **modulated by another function**. What can you infer about the modulating function?

**Observe:** (i) The real part of the transform in (b) looks like shifted sinc functions.

\[ \implies \text{The modulating (or multiplying) function looks like shifted } \delta \text{-functions under Fourier transformation.} \]

Candidate functions are therefore \( \cos(2\pi 5x) \) ad \( \sin(2\pi 5x) \) because both transform to \( \delta \)-functions.

But, \[ \mathcal{F} \{ \sin(2\pi 5x) \} = \frac{1}{2} \left[ \delta(u-5) - \delta(u+5) \right] \in \mathbb{R}^{1} \]

And, \[ \mathcal{F} \{ \cos(2\pi 5x) \} = \frac{1}{2} \left[ \delta(u-5) + \delta(u+5) \right] \in \mathbb{R}^{1} \]

So, the modulating function is \( \frac{1}{2} \cdot \cos(2\pi 5x) \).
Figure 3:

3. Consider Figure 3. The top row shows the entirely real function $\text{sinc}(x)$, while the bottom row shows its discrete Fourier transform. State at least 2 possible reasons why there is a non-zero imaginary part for the discrete Fourier transform.

- The DFT requires "compact support" in both the sampled time or space domain and the sampled temporal or spatial frequency domain. A single point shift in the sampling window will introduce an odd component to the $\text{sinc}(x)$ sampled $\text{sinc}(x)$.
- The odd component will transform to some non-zero imaginary part by symmetry properties of the Fourier transform, also termed "leakage".

4. What is used, implicitly or explicitly, to impose compact support on sampled functions?

Short Answer: A "window" or apodizing filter.

Any of "Boxcar", "Rect", "Hamming"... etc. of specific window types is also OK.